Circle on the Road, Washington, D.C., April 13-15

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From the Math Teacher's Circles to the Math Teacher's Classroom

April 15, 2012

PROMYS for Teachers (PfT)

PROMYS for TEACHERS was founded in 1991 alongside the PROMYS program to support national efforts to enhance problem-solving and open-ended exploration in Secondary school mathematics classrooms. PROMYS engages high school teachers and students in an experience of mathematical exploration within a supportive community of their peers enriched by undergraduates, graduate students, and research faculty, all actively developing their own research careers in mathematics and science.

How do we define effective teaching?

"An effective mathematics curriculum is one that

- pays attention to students,
- is rich with mathematics, and
- finds a way of connecting the two."

–William J. McCallum, in testimony to the National Academy of Sciences

But how we implement this depends on our beliefs

- about students and how students learn, and
- about mathematics.

Beliefs about students & student learning

- All students can achieve at high levels in mathematics;
- Students can enjoy doing mathematics;
- Effective teaching requires:
  - insight into how students think/reason/learn and solve problems;
  - understanding the "meaning" of student questions and developing strategies for mining student ideas.

Beliefs about the nature of mathematics

- Mathematics is natural
- Mathematics exists independent of us
- Experience precedes formality
- Mathematics is the study of structure
- Mathematics is the art of figuring things out

Teachers need to know the math they teach – and more!

"Teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach."

– Task force on Teachers and Teacher Preparation, National Mathematics Panel 2008.

Knowing Mathematics as a Mathematician

- Examining the whole mathematical enterprise
  - as a coherent body of knowledge
  - as a way of thinking and inquiring about the world we live in
- Having *deep experience of the doing of mathematics* – e.g.
  - grappling with problems
  - building intuition
  - developing theories
  - becoming completely absorbed in mathematical activity for a sustained period of time

Teaching as an apprenticeship to a life of exploration:

. . . we thought of teaching as an art of developing in the very young curiosity, a sense of adventure, and a sensitivity of perception, as well as the mastery of basic skills. . . . the teacher's mastery of the art of teaching . . . is certainly not an obvious consequence of familiarity with factual information. . . . the teacher's performance reflected [his/her] whole intellectual outlook.

Arnold Ross

Overview of the Mathematical Practice Standards

- Attend to precision
- Construct viable arguments
- Make sense of complex problems and persevere in solving them
- Look for structure
- Look for and express regularity in repeated reasoning
- Make strategic decisions about the use of technological tools

Experience first:

It has been observed in every human activity experience comes first, and as this experience grows the need for communication motivates the development of language. Sadly enough, in our classroom practice we place language first and experience second. We worry about what we should say in order to help the student understand. By this we mean to provide the effect of experience through the use of suitably chosen words. Not unexpectedly, the effect is at best a very pale image of the real thing.

Arnold Ross

Authentic Mathematical Experience

- Teachers and mathematicians experiencing mathematics together
  - as a collaborative activity
  - alongside students
  - as an empirical science
  - as exploration
- Key Features
  - emphasis on learning and problem-solving
  - depth over breadth
  - strengthening *mathematical habits of mind*

Two types of Mathematical Experiences

- Immersion Experiences
  - *Deeply personal* engagement in mathematical ideas
  - Extended experience of struggling with mathematics
  - Recommended as foundational course (not capstone)
- School-based mathematical activities
  - Study Groups
  - Teacher Math Circles

Typical features of an immersion experience

- Immersion in new ideas
- Necessity for open communication
- Acquiring taste for hard problems
- The central role of experience
  - empirical basis of mathematical knowedge
  - personal experience as guide for exploration
- Learning good judgement in recognizing significant ideas
- Sharing ideas with others
  - in writing
  - in seminars
- Questioning answers
- Low threshold – high ceiling

PROMYS for Teachers

Distinctive Features

- Depth over Breath
- Focus on Mathematics
- Community building

EDC/PfT seminars

5 full-day academic year seminars:

- Teachers and mathematicians **Doing Mathematics** together
- Teachers come from as far as California to attend
- Location: mostly EDC, sometimes BU
- Drawing heavily from three sources of materials:
  - PCMI courses
  - PROMYS for Teachers materials
  - EDC professional development materials
- Frequent visitors
  - Mathematicians
  - Education Researchers
  - Massachusetts Department of Education

Typical features of PfT seminars

- Doing Mathematics
- Low threshold – high ceiling
- Both familiar and new mathematical ideas
- Open communication
- Acquiring taste for hard problems
- Inquiry-based
  - experimentation and observation
  - using language precisely to describe what one sees
  - using language for reasoning and justification
- Sharing ideas with others
- Questioning answers

All within the context of teaching – with other teachers (and mathematicians)

A Sample Seminar

December 9, 2011 from 8:00AM to 3:30PM (about 15 teachers)

The morning session: Teachers talking about their classroom experiences.

- Catching up
  - Informal conversation over morning coffee and bagels.
  - Comparing notes on what's going on in different districts/schools
- Teachers gave presentations about activities they have recently used in their classrooms

The afternoon session: Work together on common topic: in this case, recursively defined functions on the non-negative integers.

The Morning Session: December 9, 2011

A Lawrence Middle School Teacher talked about some problems she showed her students about sums of consecutive integers:

In how many ways can a positive integer be written as a sum of consecutive positive integers?

Experimentation: If $r(n) =$ the number of ways ..., then

$$\begin{aligned}
r(2) &= 1: \quad 2 = 2 \\
r(3) &= 2: \quad 3 = 3 = 1 + 2 \\
r(4) &= 1: \quad 4 = 4 \\
r(5) &= 2: \quad 5 = 5 = 2 + 3 \\
r(6) &= 2: \quad 6 = 6 = 1 + 2 + 3 \\
r(9) &= 3: \quad 9 = 9 = 2 + 3 + 4 = 4 + 5 \\
r(15) &= 4: \quad 15 = 15 = 4 + 5 + 6 \\
&\quad = 1 + 2 + 3 + 4 + 5 = 7 + 8
\end{aligned}$$

The Morning Session: December 9, 2011

A Boston Teacher wrote the following on the board:

$$\sqrt{(2i)^3} \cdot \sqrt{2i} = \sqrt{(2i)^3} \cdot \sqrt{(2i)^1} = \sqrt{(2i)^4} = \sqrt{16} = 4$$

and also

$$\sqrt{(2i)^3} \cdot \sqrt{2i} = (2i)^{3/2} \cdot (2i)^{1/2} = (2i)^{3/2+1/2} = (2i)^2 = -4$$

What's going on? How can $\sqrt{(2i)^3} \cdot \sqrt{2i}$ be both 4 and -4?

How can I make sense of this for my students?

The Morning Session: December 9, 2011

A Los Angeles Teacher talked about her experience doing topology at PCMI the previous summer

- Posed questions about what one gets by gluing the edges of a square following various rules;
- Showed us "Dirac's belt trick".

The Afternonon Session: December 9, 2011

Exploration of recursively defined functions

- Recurrence
  - Arithmetic and Geometric sequences
  - Use technology to generate data
- Linear Recurrence
  - Fibonacci-like sequences

$$\begin{aligned} f(0) &= 2 \\ f(1) &= 3 \\ f(n) &= 3f(n-1) - 2f(n-2) \quad \text{if } n \geq 2 \end{aligned}$$

The sequence looks like this:

$$2,\ 3,\ 5,\ 9,\ 17,\ 33,\ 65,\ 129,\ \ldots,$$