The Problem of the Blue-Eyed Islanders

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1 Introduction

This is a puzzle that I originally read on Terence Tao's blog ([3]). For this activity, you, as a participant will act as an islander. Here is the basic set-up.

There is an island upon which a tribe resides. Here is what you, as an islander, know and how you must act and make decisions:

A. Your tribe consists of people with either brown eyes or blue eyes.

B. Your religion forbids you to know their own eye color, or even to discuss the topic.

C. You look at everyone's eye color except your own–there are no mirrors of any kind. Thus, you have no way to discover your eye-color but you know exactly how many of the other tribes-people have blue and brown eyes.

D. If you discovers your eye color, your religion compels you to commit ritual suicide at noon the following day in the village square for all to witness.

E. You (and all the other tribes-people) are are highly logical and devout. You and your fellow tribes-people know that each other are high logical and devout. You and your fellow tribes-people know that each other knows that the others are high logical and devout, etc.

   For the purposes of this logic puzzle, "highly logical" means that any conclusion that can logically deduced from the information and observations available to an islander, will automatically be known to that islander.

One day, a blue-eyed foreigner visits to the island and wins the complete trust of the tribe. One evening, he addresses the entire tribe to thank them for their hospitality. However, not knowing the customs, the foreigner makes the mistake of mentioning eye color in his address, remarking "how unusual it is to see another blue-eyed person like myself in this region of the world."

What effect, if anything, does this have on the tribe?

2 Acting as Islanders

We will play the islander scene out several times. The islanders (participants) will be be able to collect data about the eye color of the other islanders (we will either give them a card informing them how many blue-eyed islanders they see or else they will be given a blue or brown sticker on their forehead that will identify their eye-color).

Islanders must make some logical conclusions and decisions when the number of blue-eyed islanders is 1, 2, 3, 4, etc.

3 Mathematical Arguments

Tao ([3]) suggests two solutions to this puzzle:

Argument 1: The foreigner has no effect, because his comments do not tell the tribe anything that they do not already know (everyone in the tribe can already see that there are several blue-eyed people in their tribe).

Argument 2: Suppose that the tribe has n blue-eyed people for some positive integer n. Then n days after the traveler's address, all n blue-eyed people commit suicide.

Proof. We induct on n. When $n = 1$, the single blue-eyed person realizes that the traveler is referring to him or her, and thus commits suicide on the next day. Now suppose inductively that n is larger than 1. Each blue-eyed person will reason as follows: "If I am not blue-eyed, then there will only be $n-1$ blue-eyed people on this island, and so they will all commit suicide $n-1$ days after the traveler's address." But when $n-1$ days pass, none of the blue-eyed people do so (because at that stage they have no evidence that they themselves are blue-eyed). After nobody commits suicide on the $(n-1)^{\text{st}}$ day, each of the blue eyed people then realizes that they themselves must have blue eyes, and will then commit suicide on the n^{th} day. □

Which argument (if any) convinces you?

4 The Mathematics

The logic at work here is the concept of *Common Knowledge* ([5] and [2]). Here is some text from the cited Wikipedia page:

> What's most interesting about this scenario is that, for $k > 1$, the outsider is only telling the island citizens what they already know: that there are blue-eyed people among them. However, before this fact is announced, the fact is not common knowledge.
>
> For $k = 2$, it is merely "first-order" knowledge. Each blue-eyed person knows that there is someone with blue eyes, but each blue eyed person does not know that the other blue-eyed person has this same knowledge.

> For $k = 3$, it is "second order" knowledge. After 2 days, each blue-eyed person knows that a second blue-eyed person knows that a third person has blue eyes, but no one knows that there is a third blue-eyed person with that knowledge, until the third day arrives.
>
> In general: For $k > 2$, it is "$(k1)^{\text{th}}$ order" knowledge. After $k1$ days, each blue-eyed person knows that a second blue-eyed person knows that a third blue-eyed person knows that ... (repeat for a total of $k1$ levels) a k^{th} person has blue eyes, but no one knows that there is a "k^{th}" blue-eyed person with that knowledge, until the k^{th} day arrives. The notion of common knowledge therefore has a palpable effect. Knowing that everyone knows does make a difference. When the outsider's public announcement (a fact already known to all) becomes common knowledge, the blue-eyed people on this island eventually deduce their status, and leave.

5 Variations on the Island

Here are a few variations on the original problem.

1. What if the foreigner realizes his mistake the next day. Is there a way the foreigner can reduce the number of causalities?

2. What if the foreigner only realizes his mistake several days after his speech?

3. What if, after the foreigner's announcement, on that day and each day afterward, every islander must announce to all, their best guess of how long they think they'll stay on the island.

6 Similar Puzzles

6.1 The Barbecue Problem

This quite similar to the islander problem. See [2].

> A group is enjoying a picnic which includes barbecued ribs. At the end of the meal, some of these picnicers have barbecue sauce on their faces. No one wants to continue the evening with a messy face. No one wants to wipe her face if it's not messy (this would make him/her appear neurotic). And no one wants to take the risk of being thought rude by telling anyone else that he has barbecue sauce on his face. Since no one can see his/her own face, none of the messy picnicers makes a move to clean his/her face. Then the cook who served the spareribs returns with a carton of ice cream. Amused by what he sees, the cook rings the dinner bell and makes the following announcement: "At least one of you has

barbecue sauce on her face. I will ring the dinner bell over and over, until anyone who is messy has wiped her face. Then I will serve dessert." What happens?

6.2 Alice at the Convention of Logicians

See [6].

> At the Secret Convention of Logicians, the Master Logician placed a band on each attendee's head, such that everyone else could see it but the person themselves could not. There were many, many different colors of band. The Logicians all sat in a circle, and the Master instructed them that a bell was to be rung in the forest at regular intervals: at the moment when a Logician knew the color on his own forehead, he was to leave at the next bell. Anyone who left at the wrong bell was clearly not a true Logician but an evil infiltrator and would be thrown out of the Convention post haste; but the Master reassures the group by stating that the puzzle would not be impossible for anybody present. How did they do it?

6.3 Is Your Husband a Cheat?

This one has actually appeared on some job interviews ([1]).

> A certain town has a certain number of married couples (consisting of a husband and a wife). Everyone in the town lives by the following rule: If a husband cheats on his wife, the husband is executed as soon as his wife finds out about him. All the women in the town only gossip about the husbands of other women but no woman ever tells another woman if her husband is cheating on her. So every woman in the town knows about all the cheating husbands in the town except her own. It can also be assumed that a husband remains silent about his infidelity. One day, the mayor of the town announces to the whole town that there is at least 1 cheating husband in the town. What happens?

References

[1] My Tech Interviews, *Is Your Husband a Cheat?*, mytechinterviews.com, January 2010.

[2] Standford Encyclopedia of Philosophy, *Common Knowledge*, plato.stanford.edu, August 2007.

[3] Tao, Terence, *The blue-eyed islanders puzzle*, terrytao.wordpress.com, February 2008.

[4] Twofold Gaze, *In the Long Run We Are All Dead*, In the Long Run We Are All Dead, November 2009.

[5] Wikipedia, *Common Knowledge (logic)*, en.wikipedia.org.

[6] Wikipedia, *Induction Puzzles*, en.wikipedia.org.

[7] Wikipedia, *Prisoners and Hats Puzzle*, en.wikipedia.org