Fairfield County Math Teachers' Circle

Dominoes, Problem Solving and Graph Theory

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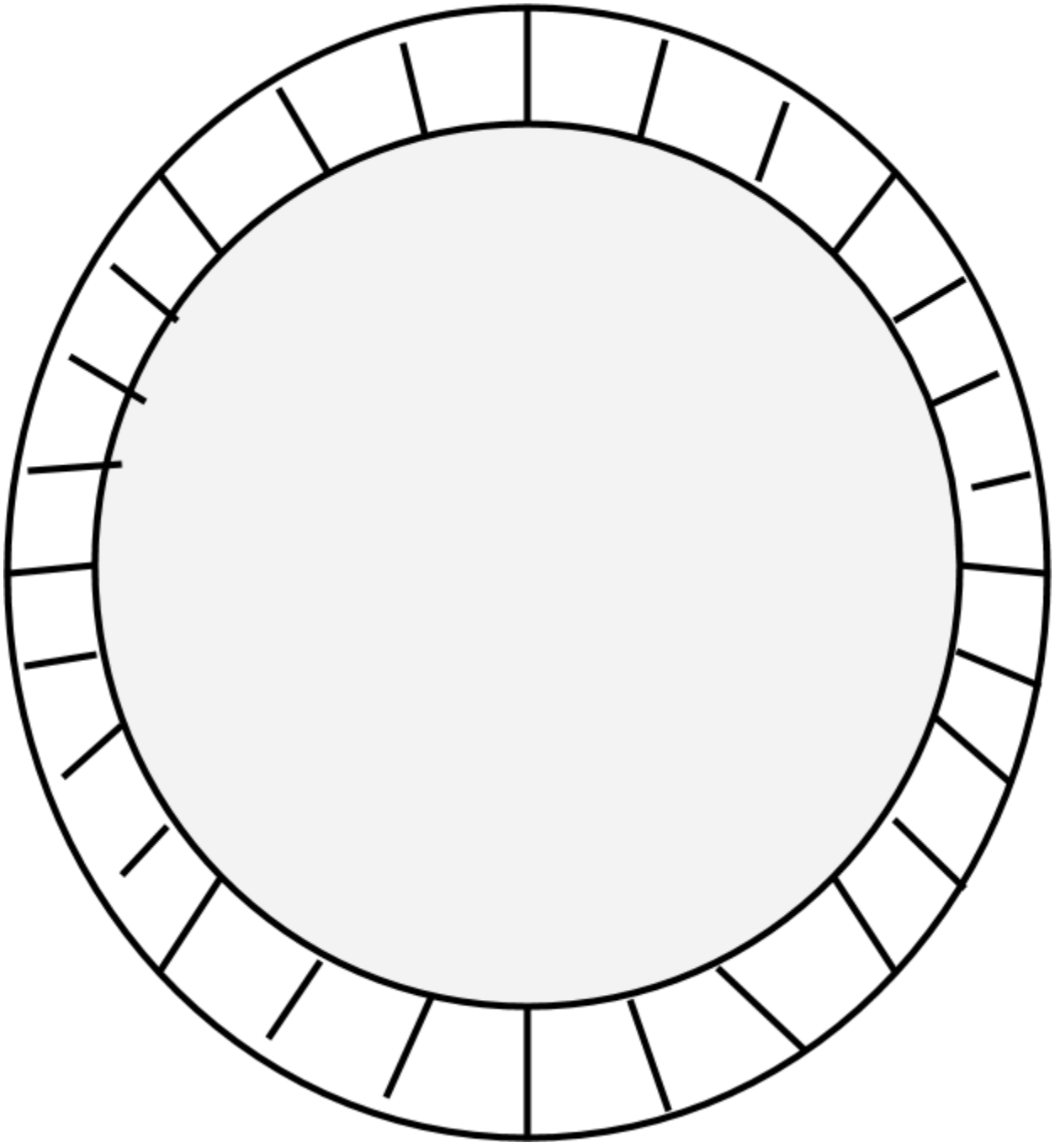
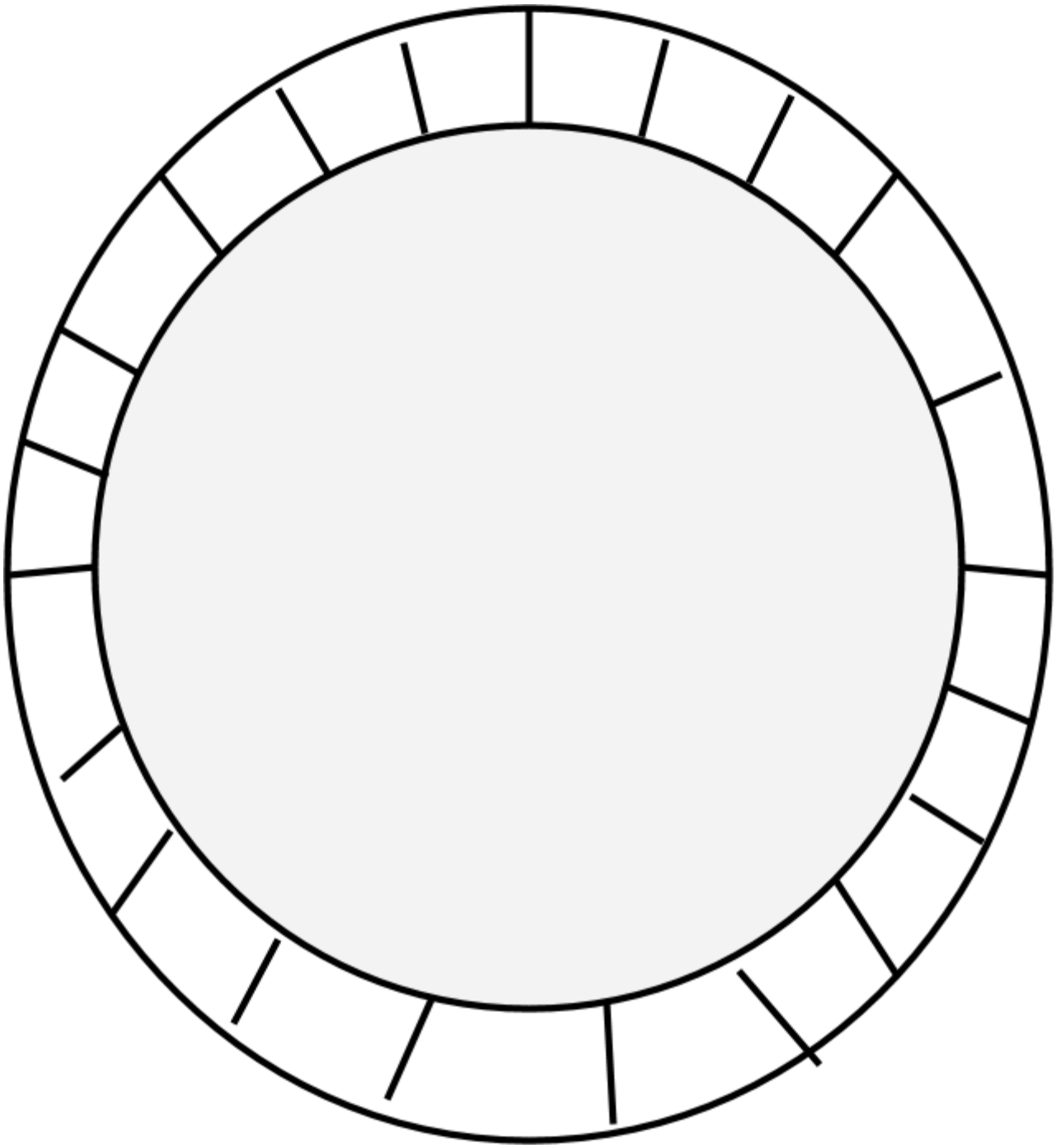
1. Can you put all 28 dominoes in a single circle, using the usual domino rules? If so, write in the dominoes in the circle below. If not, explain why not.



2. Remove all dominoes with 6 pips (dots). Can you put the remaining 21 dominoes in a circle? If so, write them in the circle. If not, explain why not.



3. Now, try to make smaller domino circles and complete the table below:

n=highest # of pips	**Dominoes**	Can you make a domino circle? **(Yes or No)**
1	00, 01, 11	
2	00, 01, 02, 11, 12, 22	
3	00, 01, 02, 03, 11, 12, 13, 22,23, 33	
4	00, 01, 02, 03, 04, 11, 12, 13, 14, 22, 23, 24, 33, 34, 44	
5	Too many to list...	
6	Too many to list...	

Discuss with your group and write down your observations.

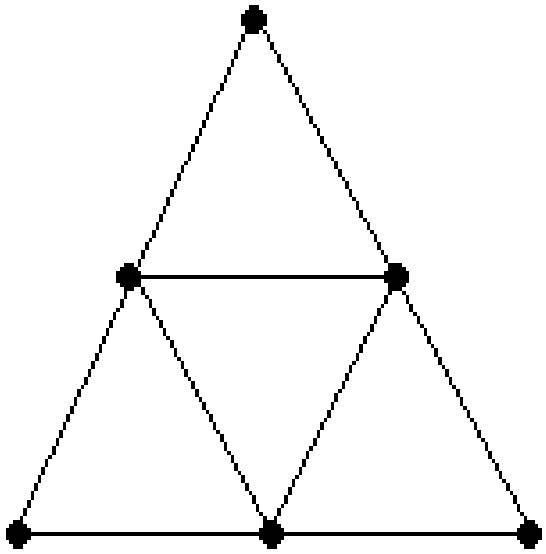
1. Do you notice a pattern?

2. Can you make a "conjecture" that works for any "n"?

3. Why do you think this pattern holds?

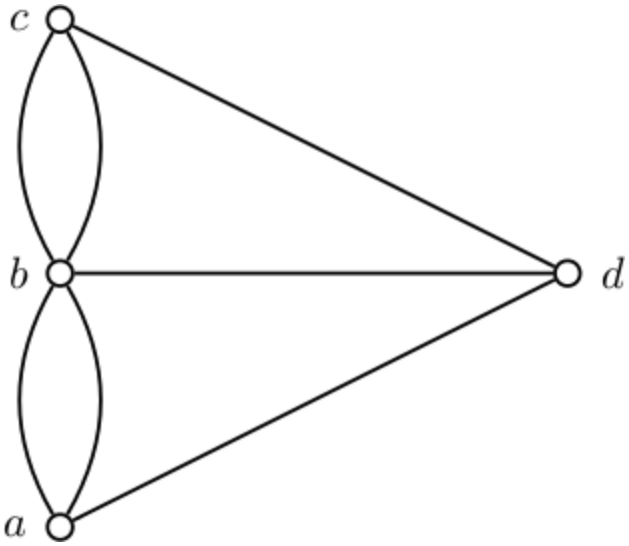
4a. **Graphs** are networks of vertices (points) and edges (line segments).

Q1: In which figures can you retrace the edges exactly once without lifting your pencil?
Q2: In which figures can you also start and end in the same place?



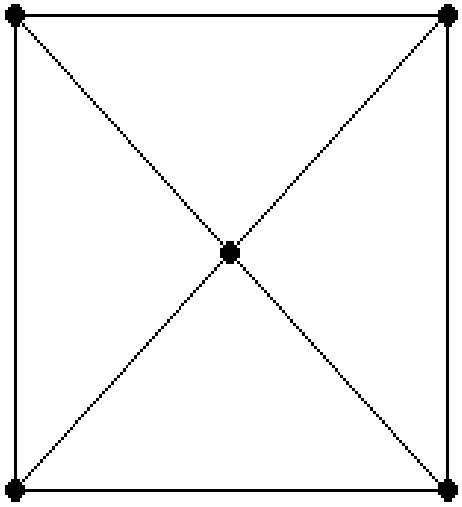
A

Q1: Yes or NO
Q2: Yes or NO



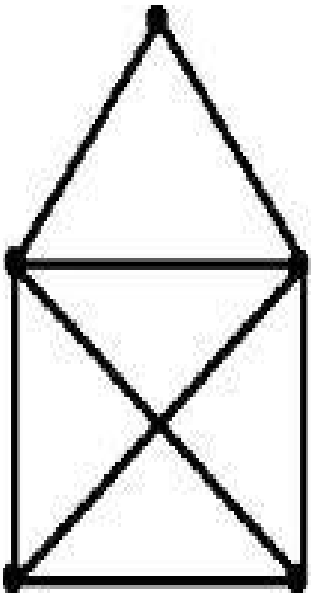
B

Q1: Yes or NO
Q2: Yes or NO



C

Q1: Yes or NO
Q2: Yes or NO



D

Q1: Yes or NO
Q2: Yes or NO

4b: **Your turn to explore:** Draw several of your own graphs below. For each one, answer Q1 and Q2.

4c. Discuss with your group and write down your observations.

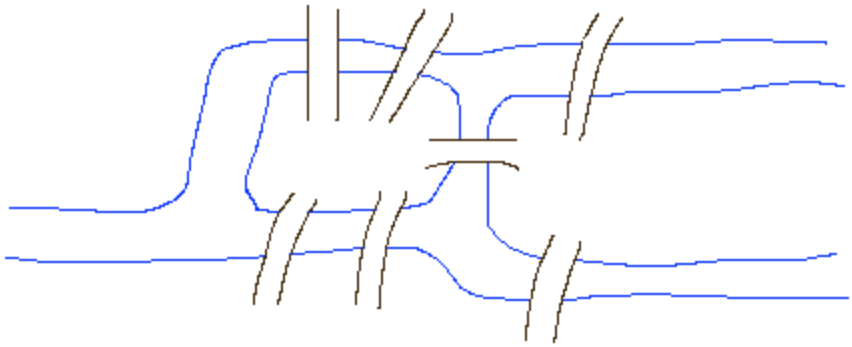
1. Did you find a pattern for which graphs can (and can't) be drawn without lifting your pencil?

2. Did you find a pattern for which graphs can be drawn without lifting your pencil, and start and end in the same place?

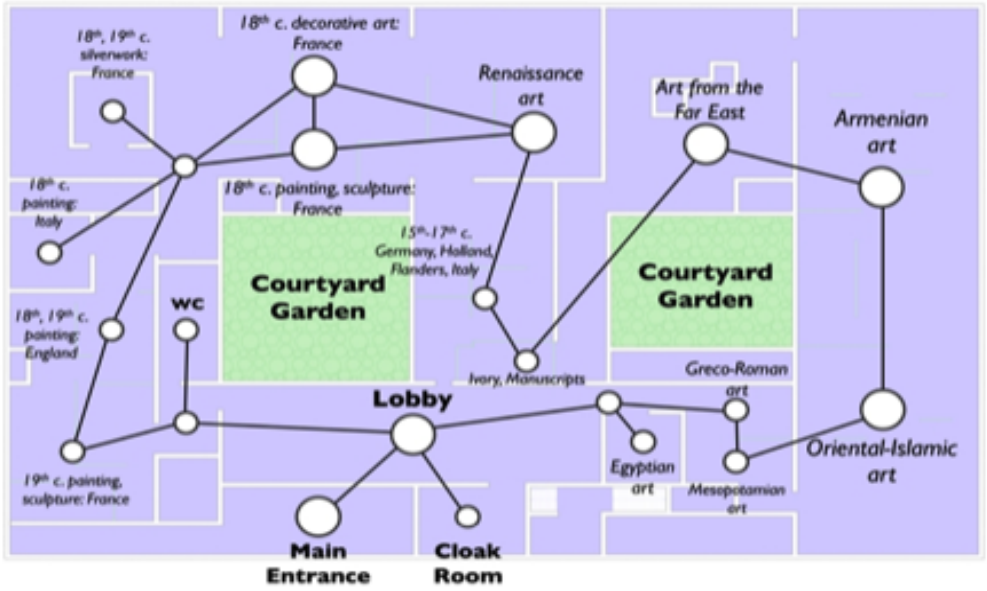
3. Did you notice any other patterns, or reasons for the patterns above?

5. Some Applications of Graph Theory

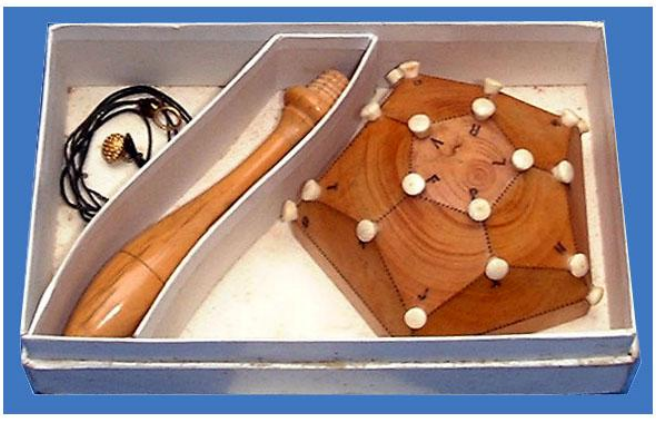
1. *The Bridges of Königsberg:* Königsberg (now Kaliningrad) was a city in Prussia on the Pregel River. Can you cross over all 7 bridges in town exactly once? (*Hint: This is related to Graph B. Can you see how?*)

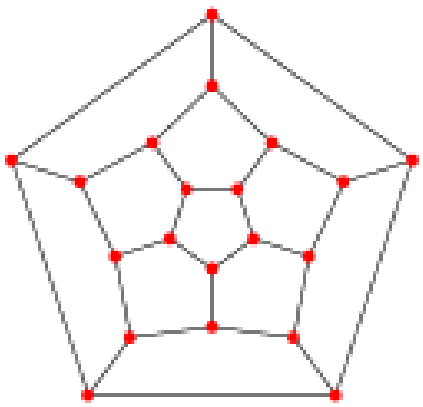


2. Below is a floor plan of a museum. Can you see all the exhibits without retracing your steps?

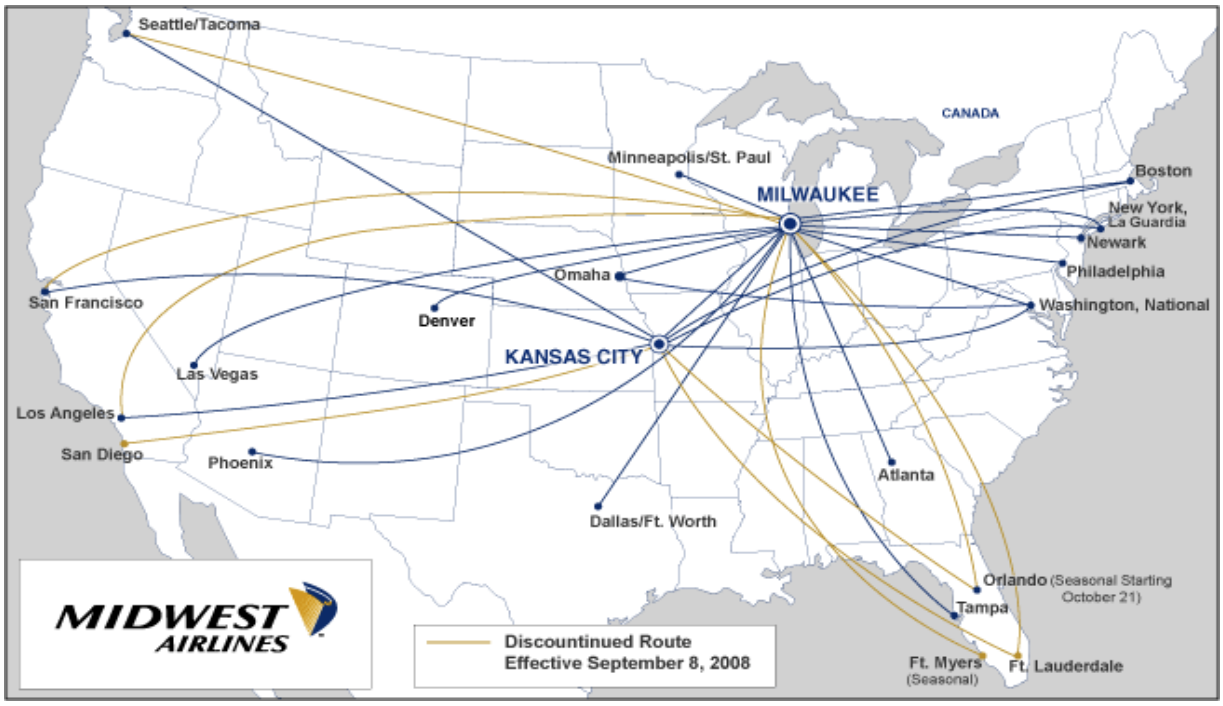


3. The "Traveler's Dodecahedron," a game invented by William Rowan Hamilton, a 19th century Irish mathematician: Can you find a path to all the cities *without* revisiting a city?





4. Airline route maps show the flight paths of airplanes from one destination to another.
Problem: What is the most efficient to visit every city exactly once?



5. A **Facebook** graph of friendships: Create your own at: https://**apps**.facebook.com/**friendsgraph**/