

Newark Area Math Circle

Grades 5-8

April 13, 2013

The Euclidean Algorithm and Linear Diophantine Equations

1. Use the Euclidean Algorithm to find the greatest common divisor of each pair of numbers
 - a. 2001 and 714
 - b. 10000 and 1001
2. Given an unlimited supply of water, an unmarked 27-liter jug, and an unmarked 21-liter jug,
 - a. Can we measure out 1 liter?
 - b. What is the smallest amount we can measure?
 - c. How exactly can we do that? Also, verify by giving a demonstration.
3. Use the Euclidean Algorithm to find the GCD of each set of integers
 - a. 300, 105, and 330
 - b. 204, 68, and 285
 - c. 1002, 750, 20004
4. For each equation, find 3 solutions and plot these solutions on the Cartesian plane
 - a. $3x + 4y = 7$
 - b. $13x + 15y = 1$
 - c. $7x + 12y = 5$
 - d. $4x + 10y = 6$

5. Find all the solutions for each of the above linear Diophantine equations by noting that:

If (x_0, y_0) is any solution of the equation $ax + by = c$, and if $\gcd(a, b) = d$, then the general solutions are given by $x = x_0 + \left(\frac{b}{d}\right)n$ and $y = y_0 - \left(\frac{a}{d}\right)n$, where n is any integer. We also note that the equation has no solution if d is not a divisor of c .

6. Find all the solutions (if any) for each equation
 - a. $4x + 6y = 9$
 - b. $6x + 15y = 8$
7. Mary wants to form 97 cents in postage stamps using only 12-cent stamps and 9-cent stamps. How many of each of these stamps can she buy?