

Santa Ana Math Club
December 11th, 2010

Name: _____

School: _____

Grade: _____

Introductory Fractions and Decimals:

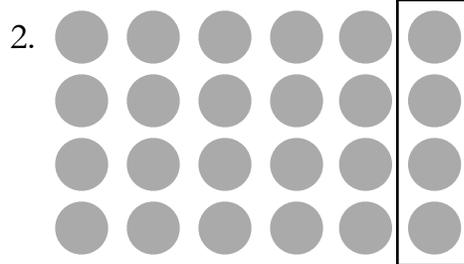
Do this for each problem:

What fraction of the circles are in the box? Simplify.

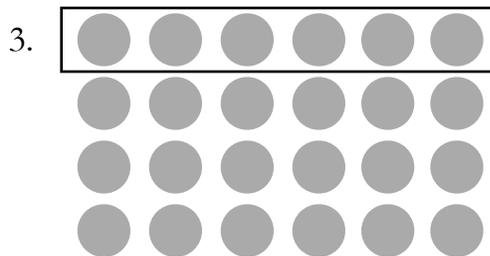


1. a) What fraction is this? _____

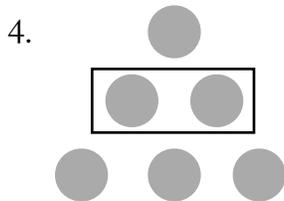
1. b) What decimal is this? _____



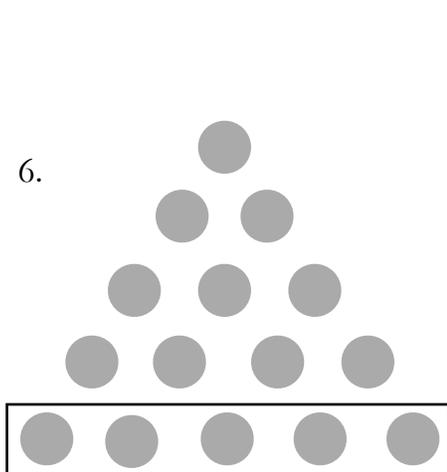
2. What fraction is this? _____



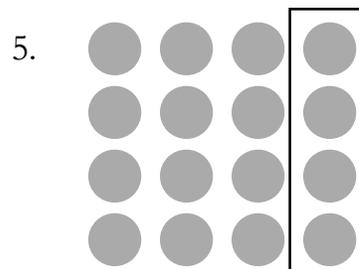
3. What fraction is this? _____



4. What fraction is this? _____



6. What fraction is this? _____



5. a) What fraction is this? _____

5. b) What decimal is this? _____

5. c) What problem represents the same fraction as this one? _____

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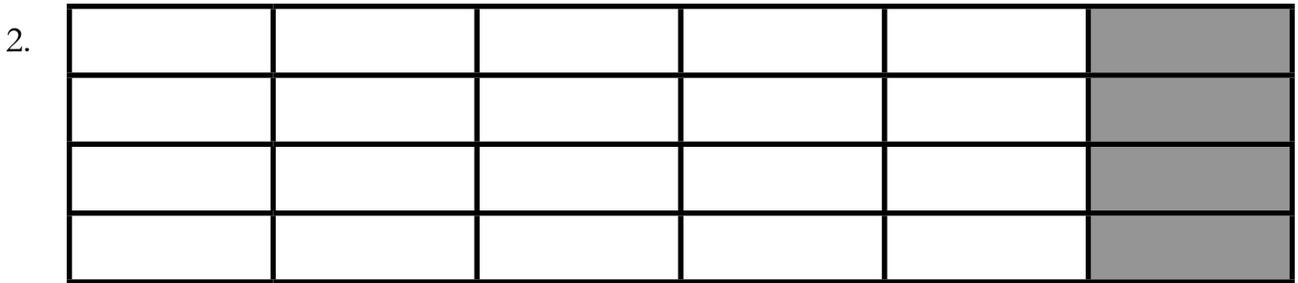
Do this for each problem:

a) What fraction of the box is shaded? Simplify.



a) _____

b) _____

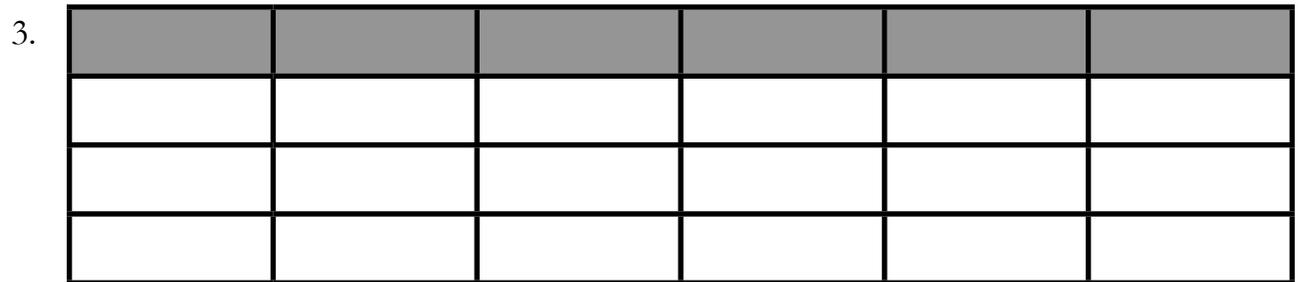


or



a) _____

b) _____



or



a) _____

b) _____

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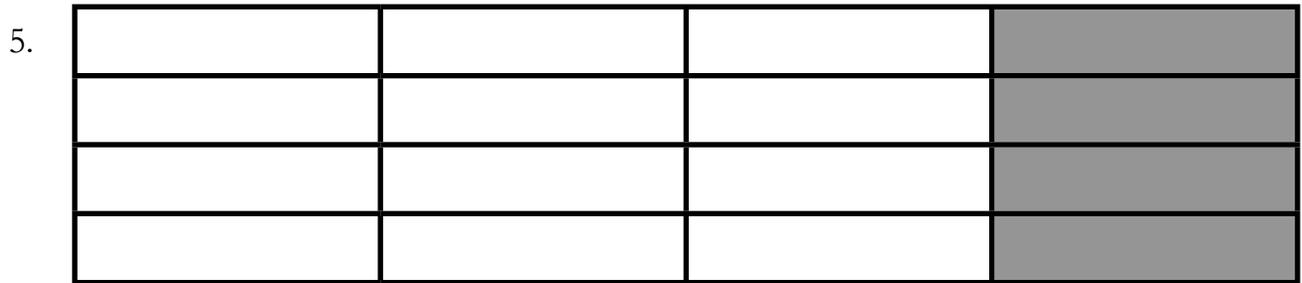
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a) _____

b) _____



or



a) _____

b) _____



a) _____

b) _____

Note:

Did you notice the similarity? The problems on page 1 and the problems on the last two pages are the same! They are different ways of representing the problems, so pick whatever way you like when working with fractions.

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Do this for each problem:

a) What fraction of the box is shaded? b) What is this decimal? Simplify.



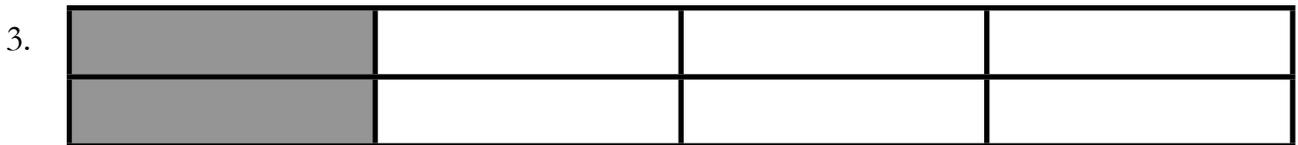
a) _____

b) _____



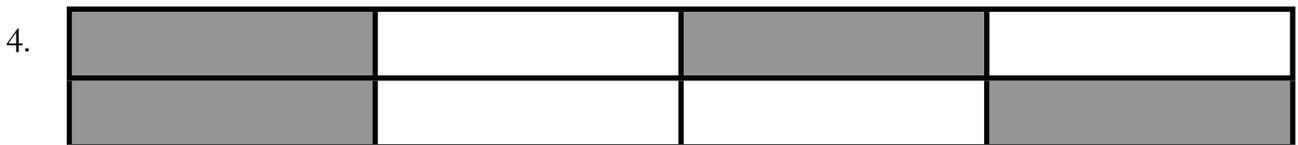
a) _____

b) _____



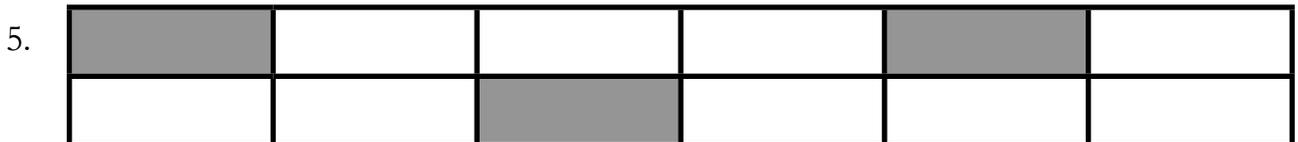
a) _____

b) _____



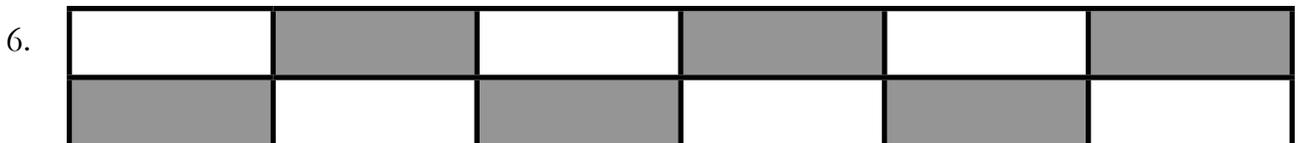
a) _____

b) _____



a) _____

b) _____



a) _____

b) _____

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1. Which number is the largest? 1.513, 1.524, 1.506, 1.507
2. Last weekend, I watched three science fiction movies, four mystery movies, and one adventure movie. What is the fraction of the movies I watched over the weekend were science fiction?
If you want to, use the bar below to help you with the problem.

3. Which number is largest? $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{5}$,
4. Which number is largest? 0.100, 0.1, 0.10000000, 0.100000001
5. Which number is smallest? 0.00099999, 0.10191, 0.0000000001111119292929
6. The product of $\frac{1}{3}$ and $\frac{2}{3}$ is?
7. The product of $\frac{1}{4}$, $\frac{1}{4}$, and 16 is?
8. What is the **reciprocal** of each of these numbers:
- a) 4 b) 3 c) 1 d) -1 e) $\frac{3}{7}$

VOCAB:

RECIPROCAL the number that you multiply to a number to get a product of 1.

Example: $\frac{1}{2} \times 2 = 1$

Practical definition: the number flipped upside down

$$\text{Reciprocal} = \frac{1}{\text{number}}$$

$$\text{Example 2: } \frac{2}{3} \times \frac{3}{2} = 1$$

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9. I have \$0.20 in nickels and \$0.90 in dimes. How many coins do I have total?
10. The sum of $\frac{1}{3}$ and $\frac{1}{2}$ is half of the fraction I am thinking of. What is the fraction? Give the answer as an improper fraction in its simplest form.
11. This morning, my sister and I got 10 dollars. I got half of the total money and my sister took the rest. I spent $\frac{1}{5}$ of the money by the end of the day. How much money do I have remaining now?
12. I got 10 dollars in the morning. I spent \$2.50. I then spent \$1.05. a) How much money did I spend?
b) How much money do I have left?
13. My sister is a triathlete. She competes in the triathlon, which consists of bicycling, running, and swimming. **(Note: In case you don't know this, you should because this frequently pops up on tests so do not be confused)**
- a) The entire race is 15 kilometers. If each part is equal in length, then how far did she swim?
- b) The entire race is 21 kilometers. She ran $\frac{1}{3}$ of the race. How far did she run?
- c) The entire race is 24 kilometers. She ran $\frac{1}{3}$ of the race, and swam $\frac{1}{4}$. The rest she biked. How far did she bike?
- d) The entire race is 36 kilometers. She ran one-fourth of the race. The remaining length of the race was split into thirds. One-third was swimming while two-thirds was running. How far did she swim?

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14. Of the people in my homeroom, twenty people take Spanish. Of these twenty people, only 8 can speak no languages other than Spanish and English while the others can speak at least three languages. The remaining ten people in my homeroom do not take any language at school and can only speak English. What fraction of people in my homeroom speak more than two languages?
15. There are fifteen boys in a classroom. $\frac{2}{3}$ said that they like to play basketball. $\frac{5}{6}$ of the girls said that they like to play basketball. The same number of girls and boys said that they like to play basketball. How many girls are there in the class?
16. At a picnic, half of the guests were men and the other half were women. $\frac{2}{3}$ of the men brought food, while $\frac{1}{2}$ of the women did. What fraction of people did not bring food.
17. You have to read a list of books by the end of six months. You decide to read $\frac{1}{6}$ of the books the first month. The next month, you read $\frac{1}{5}$ of the remaining books. The following month you read $\frac{1}{4}$ of the remaining books, and so on. The sixth month, you read 3 books. How many books did you start?
18. Convert the following decimals to simplified fractions or fractions to decimals:

Note: Sometimes it might help to reduce.

(This is an even better hint: try to make a fraction $n/100$ to get it to decimals without a calculator)

You're welcome :)

- | | | | | |
|------------|-----------------------|------------|--------------|-------------|
| a) $1/4$ | b) $0.33\overline{3}$ | c) 0.4 | d) $22/25$ | e) $3/7$ |
| a) $39/78$ | b) $4/11$ | c) $4/9$ | d) $3/9$ | e) $27/30$ |
| a) $3/12$ | b) $39/50$ | c) $17/25$ | d) $16.5/33$ | e) $47/188$ |

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19. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{1}{5}$ is what?

20. **Challenge:** In a school, half of the teachers are men and the other half are women. Two-thirds of the men are married, while three-fifths of the women are. What fraction of the teachers in the school are married?

21. **Challenge:** The price of a coat has been reduced 20%. For today's sale, we only pay 75% of the reduced price.

a) What percentage of the original price does it now sell for?

b) If the price was originally 100 dollars, what does it sell for now?

c) If the price is 90 dollars now, what was the original cost?

d) Create a situation of two or more price reductions that would lead to the percentage in part a. If you have time, make samples with even more price reductions that equal the percentage in sample a. Or, create a percentage and try to create a problem like number 21 that reaches that percentage.

Understanding Fractions and Decimals Visually

Orange County Math Circle
<http://www.ocmathcircle.org/>
Alex Zivkovic
December 11, 2010

1. Introduction

The activity described below was first implemented in the Orange County Math Circle's Santa Ana Math Club program. The problem set was delivered to students in grades 4 through 7, and contains material that is meant to challenge students at varying math levels.

Please review these instructional pages to see how a formal lesson plan can be developed from our problem set. This packet can be used to accomplish a variety of things, so lesson plans targeted at varying mathematical levels can be created.

The **first** is for students that have little to no understanding of fractions (grades 3 and below). For this group of students, this lesson can help explain the concept of fractions by presenting fractions visually. Supplements for this would include "mancala beads" or beans of varying colors.

The **second** application is for advanced math students in upper elementary (grades 4 through 6), for whom the concept will only be reinforced by seeing what fractions truly stand for. Many students who excel in school mathematics struggle with the concept of fractions in the practical world. This lesson plan seeks to help these students see how and why fractions are what they are, instead of simply teaching rote mechanics to perform operations with them. Additionally, these packets are useful in demonstrating how and why fractions can be simplified.

2. Visually grouping parts and relating them to the whole (Page 1)

First Application

For these students who have never worked with fractions or have only recently been exposed to fractions, begin by reviewing the concept of fractions. Fractions are a way of expressing the relative size of a part of a group to the whole.

Ask the class to share what they know about fractions. If any of the students are familiar with fractions, ask them to share how they are expressed (notation) as well. Have a class discussion or several small group discussions, depending on the class size.

Now, have the class work on the first page of problems. Ask them to work individually or in groups. Ask them to draw rectangles of the same size around the remaining circles for each of the problems. This will create X number of groups. Ask them to count the groups up. This is the denominator of the simplified fraction.

Another activity that students can do is to map out the same problems with mancala beads or beans of different colors. Ask them to play around in grouping the beans in similar manners as to what the rectangles do for the problems on the sheet.

Second Application

The same activity as above can be done, however in this case, also demonstrate that the simplified fraction that you end up with is equal to the fraction you would get when counting the beads. Example, on problem number 2. The fraction is $\frac{1}{6}$. However, if you counted the circles individually (not in groups), the fraction would be $\frac{4}{24}$, which simplifies to $\frac{1}{6}$.

3. Using rectangles as units (Pages 2 and 3)

First Application

Do not tell students that these problems are the same. Page one and page two/three feature the same problems expressed in different ways.

First see if students can solve the problems without your assistance. Many should be able to apply the ideas of page one to page two.

Second Application

There is a note at the bottom of the page saying that the problems are the same. Ask them to demonstrate why and tell them to find connections between how the problems were organized.

4. Extra practice to secure the “rectangles” fraction idea (Page 4)

First and Second Application

Students should be asked to solve the problems. Suggest to them to use highlighters to “move” the shaded parts over into groups. Older students will be able to solve this more easily than the younger ones, so help guide them through the process.

5. Extra practice to connect decimals and fractions (Pages 5 to 8)

First Application

Work with students closely to guide them through the problems. You should judge their level and cut off the pages accordingly. The problem set was created so that the latter problems can be removed without cutting away any significant subject matter. The recommendation for younger students is to attach all pages except 8.

Some of the problems will be easy, however the harder ones (such as 18) should be done in groups to ease the workload.

Second Application

Ask students to work on the problems individually. Problems such as 18 should be paid special attention. These problems will help them connect fractions and decimals and will allow them to increase their speed in math competitions if they can easily switch between them.

Looking for patterns (such as being able to divide the numerator and denominator) will help students notice these patterns in their future work and increase their speed.

Problem number 21 represents a fun challenge that can perhaps be made into a project for individual or group work.