

# Frieze Patterns

“Circle on the Road”

David W. Brown

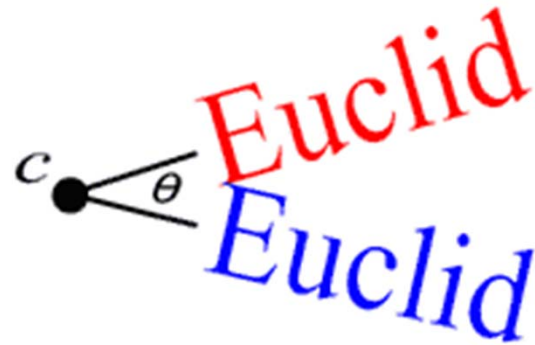
San Diego Math Circle

# Euclidean Isometries

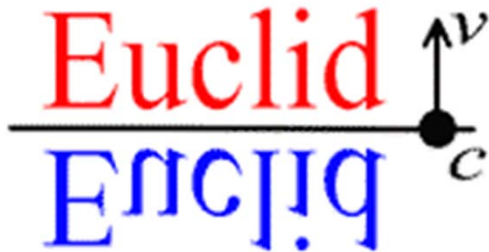
Translation



Rotation



Reflection



Glide Reflection



# Identifying Isometries

Preserves hands?

Yes

No

Yes

**Rotation**

**Reflection**

**Fixed  
Point?**

No

**Translation**

**Glide**

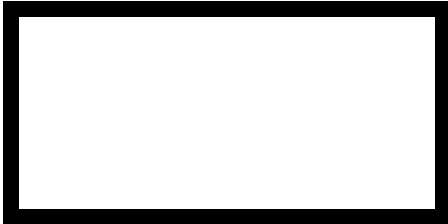
	Yes	No
Yes	<b>Rotation</b>	<b>Reflection</b>
No	<b>Translation</b>	<b>Glide</b>

# What is a Frieze Pattern?

- A frieze pattern is any two-dimensional pattern that replicates sequentially along a line.
- Frieze patterns are found naturally on ribbons, wrapped around cylinders, bordering rooms, fences, striping on roads, framing around other objects, etc.

# Creating / Dissecting a Frieze

**Identify the frieze unit**

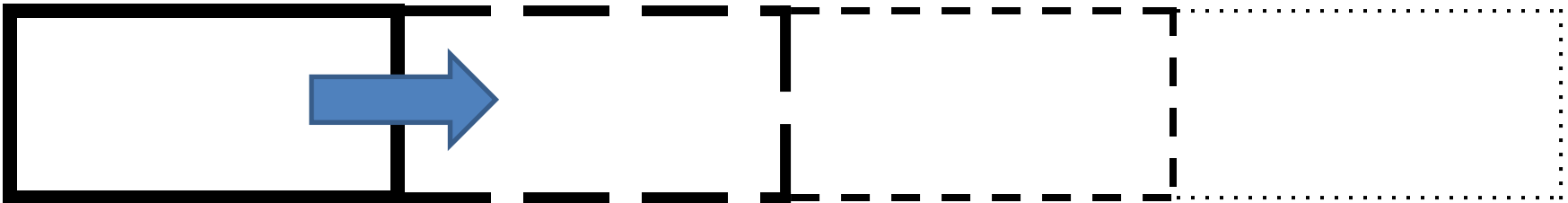


Questions:

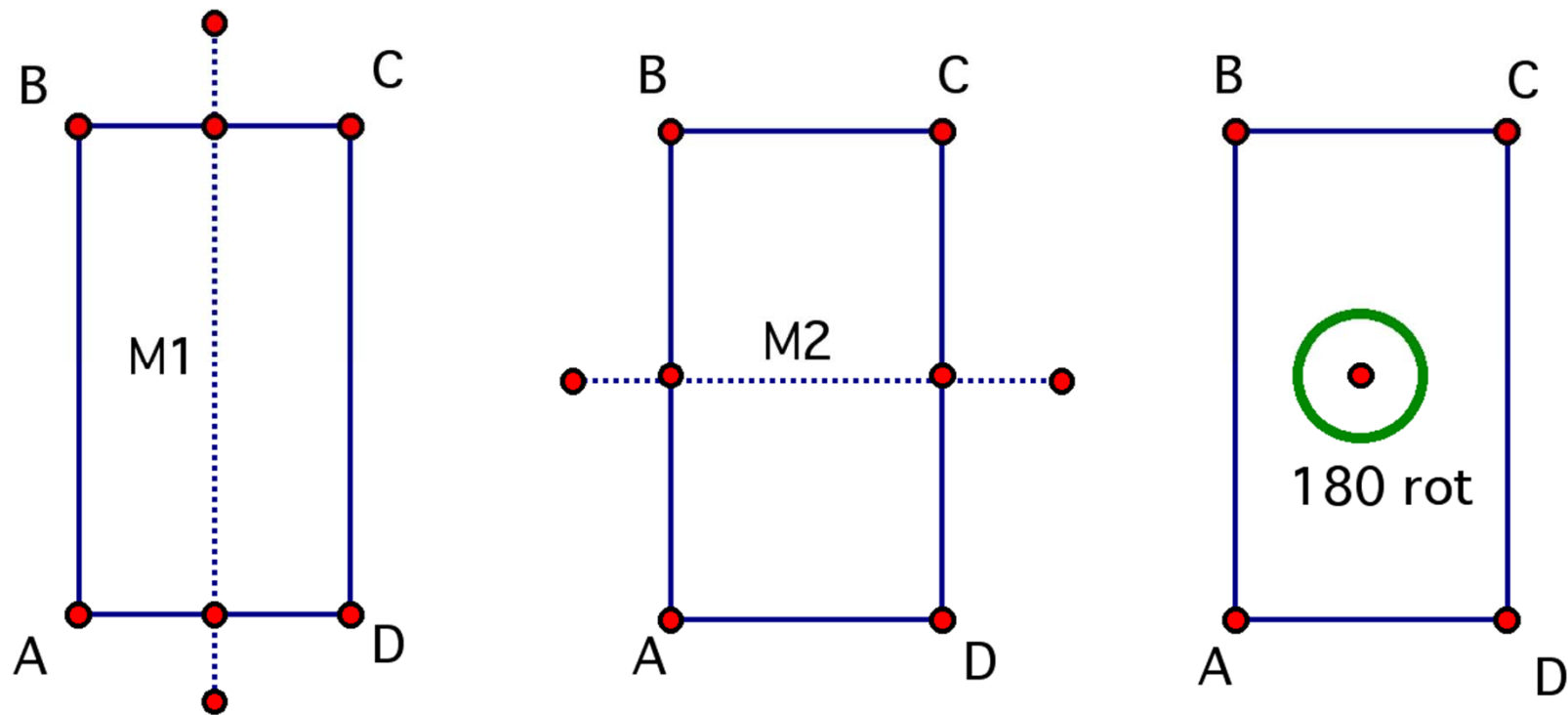
What are the properties of the frieze unit?

How do we replicate the unit?

**Replicate the unit**



The properties of frieze patterns are rooted in the (point) symmetries of a very simple object, the rectangle (not a square).



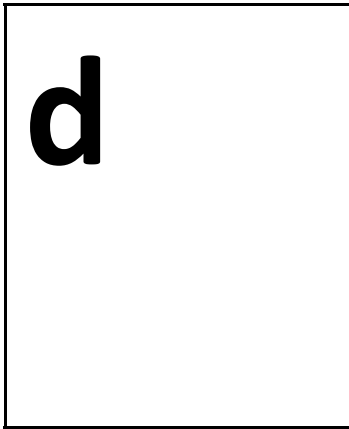
# How to build a frieze pattern

- Start with motif that will be replicated in spatial sequence to create the frieze.
- Basic questions:
  - How many classes of motif are distinguished by symmetries?
  - How many ways can the basic motif be replicated “in spatial sequence”?

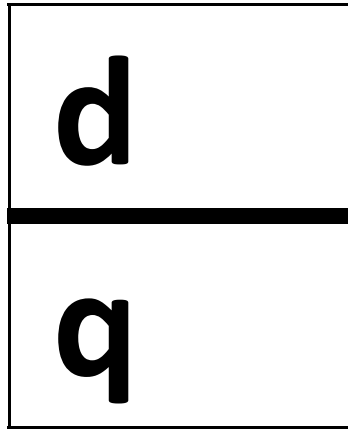
- What are the spatial symmetries a frieze motif may display?
- All frieze motifs occupy a rectangle, so the relevant symmetries are those spatial operations that map rectangles to rectangles:
  - Vertical mirror reflection
  - Horizontal mirror reflection
  - $180^\circ$  rotation



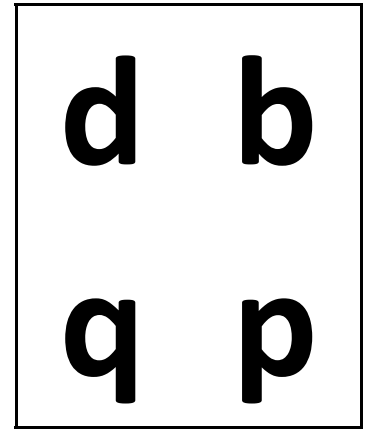
# The possible motif types



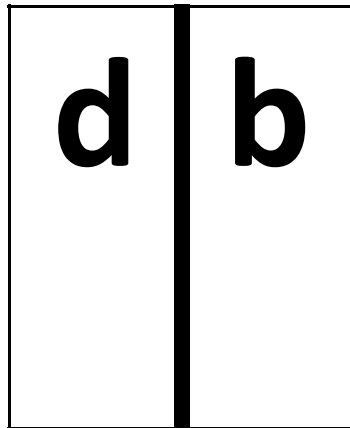
No symmetry



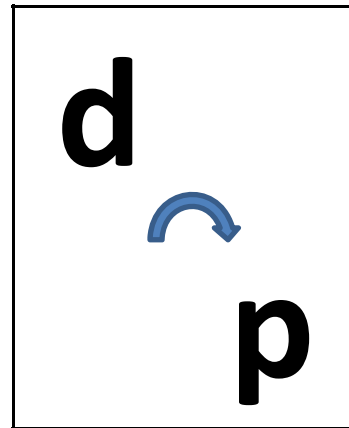
Horizontal  
Reflection



Combination



Vertical  
Reflection



180°  
Rotation

# Translation

- The basic motif can be translated into the next position to generate a frieze:

Non-symmetric one-element motif

“hop”

11

**d**

**d**

**d**

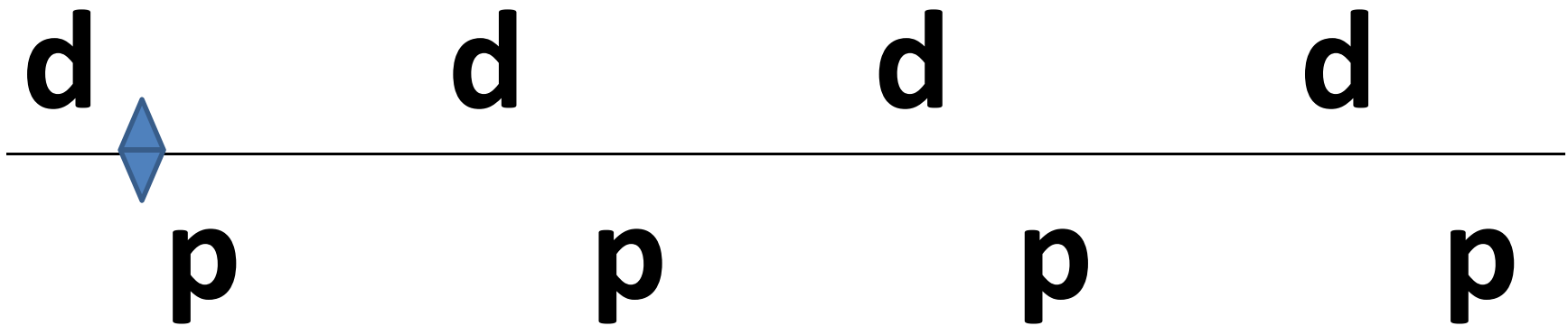
**d**

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Two-element motif with 2-fold rotational  
symmetry

“dizzyhop”

12



Two-element motif with horizontal mirror

“jump”  
1m

**d**

**d**

**d**

**d**



**q**

**q**

**q**

**q**