MODULAR ARITHMETIC, ISBN NUMBERS, and CHECK DIGITS

Definition: If n is a positive integer, then

$$\boldsymbol{a}\ mod\ \boldsymbol{n}$$

is the remainder **r** obtained when a is divided by n according to the division algorithm and it is denoted by

$\boldsymbol{a}\ mod\ \boldsymbol{n} = \boldsymbol{r}$ *or* $\boldsymbol{a} \equiv \boldsymbol{r}\ mod(\boldsymbol{n})$.

Examples

a) $55\ mod\ 4$
   By the division algorithm, $55 = 13 \cdot 4 + 3$ so the remainder of 55when divided by 4 is 3.That is, we can say $55 \equiv 3\ mod(4)$ or $55\ mod\ 4 = 3$.

b) $245\ mod\ 35$
   We have $245 = 7 \cdot 35 + 0$, and therefore $245\ mod\ 35 = 0$.

International Standard Book Number (ISBN) and Check Digit

The ISBN uses a weighted sum of the first nine digits in the number to create check digit, which is used to detect single-digit errors or the transposition of two digits. Each digit is weighed, by multiplying it by a constant, according to its position in the number. The weighted sum is found by the following formula.

For the 10-digit ISBN number $d_1 d_2 d_3 d_4 d_5 d_6 d_7 d_8 d_9\ d_{10}$, the check digit d_{10} is determined by the equation

$$10d_1 + 9d_2 + 8d_3 + 7d_4 + 6d_5 + 5d_6 + 4d_7 + 3d_8 + 2d_9 + 1d_{10} \equiv 0\ mod(11)$$

When the check digit is 10, it is denoted by an "X."

ACTIVITY

1. Is 0-816-3805-9 a valid ISBN number?

2. Calculate the check digit for each of the following partial ISBNs.

   a) 0-24-361427

   b) 3-92-392206

3. In the following ISBN, a scanner made a single-digit error, but the check digit is correct. Correct the error in as many ways as possible.

   2-08-152852-X

4. In the ISBN 4-31-011690-7, a bookstore employee made an adjacent transposition error, but the check digit is correct. Correct the error. (Hint: You might need to change two pairs)

Since 2007, ISBNs have contained 13 digits. For the 13-digit ISBN number $d_1d_2d_3d_4d_5d_6d_7d_8d_9\,d_{10}d_{11}d_{12}d_{13}$, the check digit d_{13} is determined by the equation

$$1d_1 + 3d_2 + 1d_3 + 3d_4 + 1d_5 + 3 + 1d_7 + 3d_8 + 1d_9 + 3d_{10}$$

$$+1d_{11} + 3d_{12} + 1d_{13} \equiv 0\ mod(10)$$

A 10-digit ISBN code can be converted to a 13-digit system by adding "978" prefix to the existing number and calculating the new checksum using the ISBN-13 algorithm given above.

5. Convert 0-07-037393-0 to 13-digit ISBN.

6. Suppose a four-digit code is created for each person in a group so that the fourth digit is assigned to make the sum of all four digits divisible by 8. The digits 0 through 8 may be used.

a) Is 2934 a valid identification number under this system?

b) What would the forth digit have to be to make 861 a valid four-digit identification number under this system?

c) How will a four-digit identification number be recognizable as an invalid number?

d) If a single-digit error is made in the number 7522, how will the error be detected?

e) Under what circumstance will a single-digit error not be detected? Give an example.