

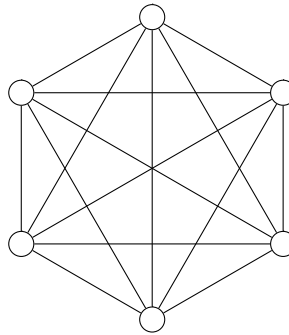
## Kayles

Prof. Shahed Sharif

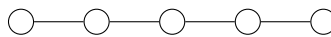
This session is focused on a 2-player game called *Kayles*, which we'll play on a game board called a *graph*. A graph is simply a bunch of circles connected by lines; the circles are called *vertices* and the lines are called *edges*. When we draw the lines, they don't have to be straight—all we care about is if two vertices are connected by a single edge or not. One each turn, a player can *either* shade in one circle, or shade in two connected circles. The player to shade in the last circle wins.

As mathematicians, the object is to determine if player 1 or player 2 can win with perfect play, and what that perfect strategy is.

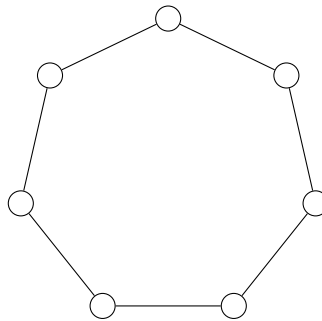
1. Start by drawing some random graphs, and playing Kayles on them to get a sense for how the game works.
2. A *complete* graph is one where every pair of circles has an edge between them. Try playing Kayles on a complete graph with 6 vertices, shown below. What about the complete graph with  $n$  vertices?



3. Now suppose there are  $n$  vertices arranged in a line. Who can win, and how?



4. What if there are  $n$  vertices connected in a circle?



5. Now suppose there are two groups of vertices, the “left” vertices and the “right” vertices. Every left vertex is connected to a right vertex, but no left vertex is connected to another left vertex, and ditto for the right vertices. Can you come up with a winning strategy?

