

## (K)not Symmetry ...

1. When something is rigid, such as a square or a triangle, it is easy to find all of its symmetries. This is not the case for knots. Imagine that the following figure was made from a loop of rope. Does it have any symmetry?



Figure 1: The  $5_2$  knot.

2. A symmetry is a rigid motion of space that carries points with rope to points with rope. How many symmetries does the loop below on the left have? What about the loop below on the right? (Don't forget to count the symmetry that does not move anything.)

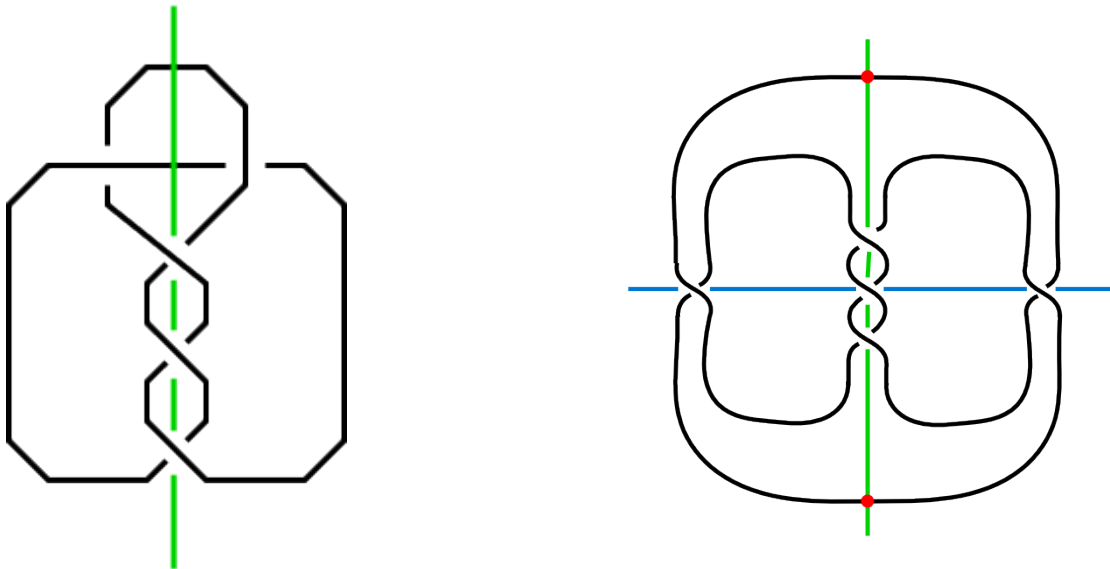


Figure 2: Full symmetry of the  $5_2$

3. Show how to deform the loop from problem 1 until it matches the the loop on the left and then the loop on the right from problem 2.
4. Play the (k)not symmetry game with a friend and the knots on the attached page (or make up and use your own knots.) To play the game, rearrange the knot to find symmetries. Your score is the number of symmetries that you find. You can keep going with more knots, and set a goal of finding 12 symmetries.
5. What is the most symmetries you can find in one knot? What is a knot with the smallest number of symmetries?
6. Use induction to prove that every rational tangle has four symmetries. Does this help you play the (k)not symmetry game?