Laughter and Insight . . .

Consider "words" i.e. strings in the two symbols A and H. When these satisfy an associative property and the commutative property $HA = AH$, one can turn random laughs into $AAHHH$'s.

1. Use the commutative property, i.e. work in $\mathbb{Z}^2 := \langle A, H \mid AH = HA \rangle$, to put each of the following strings in $AAHHH$ form (if possible).

   (a) $HAHAA$

   (b) $HAHAHA$

   (c) $HAAHHAAHHHA$

2. Show that the following two strings are equivalent using the commutative property: $AH^3AHA^4H^5$ and $H^5A^4HAH^3A$.

Before I could see, I could not read $AH = HA$ on a blackboard, but I could see $AAH = HAA$ and $HHA = AHH$. This was still pretty darn good, so let's set

$$DG := \langle A, H \mid A^2H = HA^2, H^2A = AH^2 \rangle .$$

3. Help guide the visually impaired through problems 1 a) - c) and 2.

4. Characterize which strings in A and H are equivalent to A^nH^m in DG. Hint: Do the tiling activity first, then do problems 7 and 9 before coming back to this question.

It is helpful to complete these languages into groups by adding a symbol 1 and for every symbol X, a new symbol X^{-1} such that $1X = X1 = X$ and $X^{-1}X = XX^{-1} = 1$. Consider a language with a 1: $ABBA$ is a disco group that inspired its own language. Every word in $ABBA$ is written with As Bs and possibly 1s. The people who speak $ABBA$ do not like double letters, so they replace any double letter by 1. Thus the word $ABBA$ is equivalent to $A1A$ but this is the same as AA, which is the same as 1. In addition, they do not want their language to just turn into a bunch of babble words like $BABABABA$ (they are not sheep). Thus they also replace ABA and BAB by 1.

5. Show that BA has the same meaning as A in this language.

6. Make a dictionary of all words in $ABBA$. (On a tropical island there is a large group of people who like $ABBA$ and braids. Check out the braid activity.)

Build models to touch and feel to understand what is going on. For any one of these alphabets with relations, make a graph with the normal form of each possible word as vertices, and directed edges connecting a word W to WS whenever S is a symbol in the alphabet.

7. Build and understand the models for the following languages:
   (a) $HEX := \langle A, B \mid A^2 = B^2 = (AB)^3 = 1\rangle$
   (b) $OCT := A, B, C \mid A^2 = B^2 = C^2 = (AB)^3 = (BC)^3 = (CA)^3 = 1\rangle$
8. Kick around the graph for $SOC := \langle A, B \mid A^2 = B^5 = (AB)^3 = 1\rangle$.
9. Find the graph for $\mathbb{Z}^2$. What part of this graph corresponds to $A^2HA^{-2}H^{-1}$?
10. Find the graph corresponding to DG.