Laughter Guide

The notation in this activity may be a bit difficult for elementary school students. However the concepts are understandable. You will need to explain that $H^5 = HHHHH$. It is worth asking them to think about H^3H^5 and how that could be written using the same shorthand notation. Don't worry about the notation $\mathbb{Z}^2$ –this is standard notation for the two dimensional integer lattice. You may just tell the students that this is the name of this first "language." The notation $\langle A, H \mid AH = HA \rangle$ is just a fancy way of describing a language in which the entire alphabet is just A and H, so that words are just finite strings of these letters, e.g. $AAHHHAHA$. The expression $AH = HA$ is a rule that applies anywhere, so $AAHHHAHA = AHAHHAHA$ by using the rule on the second and third letters. (We assume associativity.) There are many parts to the activity and many surprises to be found, so it is worth encouraging students to try different parts of it. It is also worth asking what other activities they have already done.

The activity splits into several parts: Part 1 consists of problems 1, 2, 3 and 4. Part 2 consists of problems 5 and 6. Part 3 consists of problems 7 and 8. Part 4 consists of problems 9 and 10. Participants may start on any of the parts. Indeed you should try to encourage students to try parts 1, 2, and 3 independently. Part 1 is closely related to the tile activity, part 2 is closely related to the braid activity and the rational tangle dance. Part 3 is an independent construction project that ties parts 1, 2, 3 and 4 together.

Part 1 The first part problem is not too difficult, once the concept is explained – change laughter into insight by swapping As and Hs until the words have all the As and the start. Try to get the participants to do this one step at a time. The rule $AH = HA$ is natural if one considers this multiplication or addition of numbers. However there are many places where it does not make sense. **Suggest that participants compare this with "its raining cats."** It is possible that some participants will be able to do problems 3 and 4 without playing with the tiling activity, but most will not. Even if someone can solve these without the tiling activity, they should still do the tiling activity and compare with this activity. **Ask participants who are working on this to compare it with problem 1 of the tiling activity. This is one of the nicest surprises of the festival.** Once someone sees how these compare, they will be able to do much more complicated problems of the same type. Even more can be said. Part 4 of this activity gives a very nice conclusion.

Part 2 Make sure that participants know the meaning of A^3 before starting. Then see if they can guess what A^2A^3 is equal to. Have them work a few until you know they have the pattern. Then ask what A^7A^{-1} should be. Hopefully the notation will make some sense to them after this. Problem 5 will take students a bit of trial and error. Once the get problem 5, they should generalize to problem 6. **Anyone who works on this should compare it with the braid activity.** Can the students see what the language of the rational tangle dance is?

Part 3 You might start by showing the participants the SQUARE language,

$$\text{SQUARE} = \langle A, H \mid AH = HA, A^2 = H^2 = 1 \rangle .$$

The geometric shape corresponding to this language is a square. Each corner corresponds to a word in the language: 1, A, H, AH. Why are these the only possible "words"? Each oriented edge corresponds to appending a letter to the end of a word. The surface patch is just the rule in the language. What would the surface patch look like for the rule $A^3 = 1$? What is the patch for $A^2 = 1$? The language

$$\text{TOCT} = \langle A, B, C \mid A^2 = B^2 = C^2 = (AB)^3 = (BC)^3 = (CA)^2 = 1 \rangle \,,$$

is one that you can show as a model after students have thought about this problem for a while. Make sure to get the students to explain *why* they have all the words in the language.

Part 4 Problem 9, follows fairly naturally from the geometry of the SQUARE language. It also creates a checker board and a domino. This is the connection to the tiling activity. It may be possible to guess this just in part 1, especially if someone has done part one of this activity and the tiling activity. Problem 10 is a bit more difficult. The following two questions may help, but might take some interpretation.

1. Find a standard way to write the strings in DG that are equivalent to A^0H^0 as elements of $\mathbb{Z}^2$.

2. Find a normal form for elements of DG.

This should be compared with the tiling functions in the tiling activity. For more information on this topic see the papers by Conway and Lagarias [1], and Thurston [2].

References

[1] J. H. Conway and J. C. Lagarias. Tiling with polyominoes and combinatorial group theory. *J. Combin. Theory Ser. A*, 53(2):183–208, 1990.

[2] William P. Thurston. Conway's tiling groups. *Amer. Math. Monthly*, 97(8):757–773, 1990.