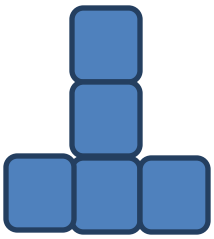
Lesson Plan: The Fourth Dimension

Opening Problem:

Ask the students if they could make a cube out of the following net. As they agree that you can indeed make a cube, ask them to explain how they know. Some might actually want to cut out the shape and demonstrate how to fold it to make a cube. Encourage them to identify which sides are adjacent and which are opposite, without actually cutting and folding the shape.



Ask the students for a different net for a cube, and again have them explain how they know their net works. Students may come up with different nets.

At this point, they're ready for the main opening question or they may come up with the question themselves:

How many different nets are possible for a cube? Generate all the possible nets. How do you know whether you have not missed some possibilities?

You might want to keep the students away from the internet since the answer to this problem is easily accessible, and encourage the students to be as systematic as possible in describing the nets. The students may group the net according to their symmetry.

The Fourth Dimension:

Ask the students what a "net" for a square would be (line segments …). Make the observation that the three dimensional cube is made from a net of two dimensional squares, and a two-dimensional square is made from a net of one-dimensional line segments.

Take the students through the analogical reasoning for a hypercube … two points joined make a line segment, two lines joined make a square, two squares joined make a cube, two cubes joined make a hypercube ….

Then view the second half of one of a series of videos called *Life By the Numbers* (http://www.montereymedia.com/science/). The video contains an interview with Thomas Banchoff, a mathematician at Brown University who has spent his career studying the fourth dimension. Following the interview with Dr. Banchoff is a segment about the artist Tony Robbin (http://tonyrobbin.net/work.htm) whose art often consists of two-dimensional representations of four-dimensional objects.

The students might then explore representations of hypercubes on the internet.

What would the coordinates of the vertices of the hypercube be in the fourth dimension?

Would Euler's formula apply in the fourth dimension?

More Questions/Activities:

- Using the same type of analogical reasoning that was used to build a hypercube, build or describe how to build the four-dimensional version of other three-dimensional shapes.

- Read *The Fourth Dimension* by Rudy Rucker (http://books.google.com/books/about/The_Fourth_Dimension.html?id=8J0djs-FK_8C) and/or answer the questions posed in that book about the fourth dimension.

- Read and discuss the book *Flatland*

- View the movies *Flatland* and *Sphereland* (and the subsequent interviews with Dr. Thomas Banchoff).

- What would a net for a hypercube look like? View the painting The Crucifixion by Salvadore Dali and discuss the cross in that painting.