The basic logic rules (De Morgan's laws) can be expressed in formal language with two propositions A and B as:

$\neg (A \lor B) \Leftrightarrow (\neg A) \land (\neg B)$

$\neg (A \land B) \Leftrightarrow (\neg A) \lor (\neg B)$

where:

$\neg$ is the negation operator (NOT)

$\land$ is the conjunction operator (AND)

$\lor$ is the disjunction operator (OR)

$\Leftrightarrow$ is a symbol meaning "can be replaced in a logical proof with"

CASE 1

1. **In this room, there is a princess and, in the other room, there is a tiger.**
2. **In one of these rooms is a princess, and in one of these rooms there is a tiger.**

One is true and the other one is false

We know that one of the signs is TRUE and one is FALSE. Let us see what happens if the first one is TRUE. ***In this room, there is a princess and, in the other room, there is a tiger***.

Princess   Tiger

But then the second one is also TRUE since it states what the first one does: ***In one of these rooms is a princess, and in one of these rooms there is a tiger.***

Since both signs can not be truthful at the same time, it becomes clear that the second sign has to be TRUE, while the first should be FALSE.

Since the second statement (sign) is TRUE, there is, in fact, one princess and one tiger in two rooms. The first one is FALSE means the first room does not have a princess or the second one does not have a tiger or both: no princes in the first room and no tiger in the second**.** Since the rooms can not be empty, the first room should have a tiger and the second one should have a princess.

Tiger   Princess

CASE 2

1. **At least one of these rooms contains a princess**
2. **A tiger is in the other room.**
either both true or both false

We know that both signs may be TRUE or both signs may be FALSE.
Let us see if both signs are FALSE. If the second sign: **A tiger is in the
other room** is FALSE it means that the tiger is not in room 1, since the
room can not be empty, this room contains a princess. That means that sign
1 (**At least one of these rooms contains a princess**) can not be FALSE.
So the two signs can not be FALSE.

Let us see what happens if they are both TRUE: **A tiger is in the other
room** means that room 1 contains a tiger.

Tiger

At least one of these rooms contains a princess is true means *that room
two contains a princess*

Tiger   *Princess*

CASE 3

1. **Either this room contains a tiger, or the other room contains a princess.**
2. **A princess is in the other room.**

either both true or both false

We know that both signs may be TRUE or both signs may be FALSE.
Let us see if both signs are FALSE. If the second sign: **A princess is in the other room** is FALSE, it means that *there is no princess in room 1 and since the room is not empty, there is a tiger in room 1*

Tiger

But then the first half of the sign 1 (**Either this room contains a tiger**) is TRUE. That makes the whole sign 1 TRUE, but we know that both signs should be TRUE or FALSE at the same time.

I both signs are TRUE. Let us look at the second sign: **A princess is in the other room**.

Princess

For the first sign (**Either this room contains a tiger, or the other room contains a princess**) to be TRUE, at least one of the two parts should be TRUE. The first part is not TRUE since we found that the princess, not the tiger, is in room 1. It means that the second half of the statement (**the other room contains a princess**) should be TRUE, and so room 2 contains a princess.

Princess   *Princess*