

Mathematics in your head – the secrets of mental math

1. Fundamentals: mental addition, subtraction, multiplication and division, and “gestimation”.

Addition:

$$42 + 3 = 45$$

$$42 + 30 = 72$$

$$42 + 300 = 342$$

$$42 + 3000 = 3042$$

Think from “left to right” and “hear the problem”

e.g. “two billion + two billion = four billion” because “two plus two is four”.

Try: “eight hundred plus four hundred” or “80 + 50”.

Adding numbers with no “overlap”

$$70 + 2 = 72 \text{ or } 500 + 43 = 543 \text{ or } 2500 + 46 = 2546.$$

On paper you might add the numbers right to left:

$$2300$$

$$+45$$

$$2345$$

2. Multiplication.

Easy: multiplication by 10, 100, 1000 (just attach zeros)

$$43 \times 10 = 430$$

$$43 \times 100 = 43 \text{ hundreds or } 4 \text{ thousands three hundred}$$

$$43 \times 1000 = (\text{simply}) 43 \text{ thousand or } 43000$$

Multiplication table

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	30	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Multiples by 9, digits add up to nine, start with a digit one less than the multiplier, e.g. $5 \times 9 = 45$

3. Solving problems by simplifying.

$$\begin{array}{r}
 432 \\
 \times 3 \\
 \hline
 3 \times 400 = 1200 \\
 3 \times 30 = 90 \\
 3 \times 2 = 6 \\
 \hline
 1296
 \end{array}$$

4. Estimation

54×7 on paper starts with $4 \times 7 = 28$, you know that the number ends up with 8

Mentally, $50 \times 7 = 350$, so the number is larger than 350, giving better information.

Getting the answer: $50 \times 7 = 350 + 4 \times 7 = 28$, $350 + 28 = 378$, $50 + 20 = 70$, and 8, 378.
 “left to right”.

5. Multiplying 11 x dd (two digit number)

$$\begin{aligned}
 23 \times 11 &= 2(2 + 3)3 = 253 = 253. \\
 53 \times 11 &= 5(5 + 3)3 = 583 = 583
 \end{aligned}$$

You try

$$81 \times 11$$

$$62 \times 11$$

What about

$$85 \times 11 = 8(8 + 5)5 = 8135 = (8 + 1)35 = 935 = 935$$

$$77 \times 11 = 7(7 + 7)7 = 7147 = (7 + 1)47 = 847 = 847$$

You try

$$58 \times 11$$

$$87 \times 11$$

$$99 \times 11$$

6. Multiplying 11 x ddd (three digit number)

$$314 \times 11 = 3(3 + 1)(1 + 4)4 = 3454 = 3454$$

$$3454 \times 11 = 3(3 + 4)(4 + 5)(5 + 4)4 = 37994 = 37994$$

7. Squaring numbers

$$15^2 = 225$$

$$25^2 = 625$$

$$35^2 = 1225$$

$$45^2 = 2025$$

$$55^2 = 3025$$

$$65^2 = 4225$$

$$75^2 = 5625$$

$$85^2 = 7225$$

$$95^2 = 9025$$

What is the pattern?

$$85 \times 85 = 8 \times 9 \text{ (next digit) } 25 = 7225$$

Use this pattern to multiply 35×35

8. Extending the pattern

$$85 \times 85 = 7225$$

$$84 \times 86 = 7224$$

$$83 \times 87 = 7221$$

$$82 \times 88 = 7216$$

$$81 \times 89 = 7209$$

What's the pattern?

$$(8 \times 9) (4 \times 6) = 7224$$

This pattern can be used for numbers with the same first digit and then second digits adding up to 10.

$$53 \times 57 = (5 \times 6) (3 \times 7) = 3021$$

You try

$$64 \times 66$$

$$48 \times 42$$

9. Multiplication table up to 20.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

Here is how to multiply numbers between 10 and 20

$$18 \times 13 = (18 + 3) \times 10 + 8 \times 3 = 21 \times 10 + 24 = 210 + 24 = 234$$

You try

$$17 \times 18$$

$$19 \times 12$$

$$14 \times 16$$

Mathematics in your head – addition and subtraction

1. Fundamentals: mental addition, subtraction

Addition:

$$42 + 3 = 45$$

$$46 + 7 = \text{since } 6 + 7 \text{ is } 13, \text{ recognize that the result will be } 50 \text{ and } 3, 53.$$

Think from “left to right” and “hear the problem”

$$73 + 16 = 73 + 10 + 6 = 83 + 6 = 89$$

You try

$$46 + 23 =$$

$$62 + 33 =$$

More difficult

$$23 + 58 = 23 + 50 + 8 = 73 + 8 = 70 + 11 = 81$$

You try

$$48 + 37 =$$

$$84 + 87 =$$

$$65 + 16 =$$

$$76 + 82 =$$

$$89 + 53 =$$

2. Adding three digit numbers.

$$400 + 567 = 400 + 500 + 67 = 900 + 67 = 967$$

$$450 + 320 = 450 + 300 + 20 = 750 + 20 = 770$$

You try

$$410 + 360 =$$

$$770 + 560 =$$

$$314 + 159 = 314 + 100 + 59 = 414 + 59 = 414 + 50 + 9 = 464 + 9 = 473$$

$$766 + 489 = 766 + 400 + 89 = 1166 + 80 + 9 = 1246 + 9 = 1255$$

Another way:

$$766 + 489 = 766 + 500 - 11 = 1266 - 11 = 1255$$

(usually easier if there is many carries)

3. Subtraction.

$$93 - 41 = 93 - 40 - 1 = 53 - 1 = 52$$

$$74 - 29 = 74 - 20 - 9 = 54 - 9 = 40 + 14 - 9 = 40 + 5 = 45$$

Another way:

$$74 - 29 = 74 - 30 + 1 = 44 + 1 = 45$$

You try

$$82 - 48 =$$

$$121 - 57 =$$

Three digit numbers

$$846 - 225 = 846 - 25 = 600 + 20 + 1 = 621$$

$$835 - 497 = 835 - 500 + 3 = 335 + 3 = 338$$

$$835 - 417 = 835 - 400 - 17 = 435 - 20 + 3 = 415 + 3 = 418$$

Finding a complement:

$$75 \quad 49 \quad 67 \quad 32 \quad 80$$

$$25 \quad 51$$

$$100 \quad 100 \quad 100 \quad 100 \quad 100$$

First digits add up to 9, second digits add up to 10, except if the number ends with zero.

$$835 - 467 = 835 - 500 + 33 = 335 + 33 = 368$$

You try

$$621 - 274 =$$

$$729 - 256 =$$

$$1234 - 567 =$$

Finding a complement:

675 849 767 232 870
325 151
1000 1000_ 1000 1000 1000

What is the pattern?

If an item costs \$8.35 and you pay with a \$10 bill, what will be the change?

What's the change for \$23.58 if you pay with a \$100 bill?

If an item costs \$13.35 and you pay with a \$20 bill, what will be the change?

If an item costs \$7.35 and you pay with a \$20 bill, what will be the change?

4. Four digit numbers

Difficult to even remember the problem, unless we need to memorize up to 6 digits:

$$1234 + 6000 = 7234$$

$$1200 + 180 = 1380$$

You try

$$3500 + 2000 =$$

$$4500 + 250 =$$

Mathematics in your head – multiplication

1. Fundamentals: mental multiplication

Two by ones

$$20 \times 7 = 7 \times 2 \times 10 = 14 \times 10 = 140$$

$$23 \times 7 = 7 \times 20 + 7 \times 3 = 140 + 21 = 161$$

$$87 \times 8 = 8 \times 80 + 8 \times 7 = 640 + 56 = 640 + 50 + 6 = 690 + 6 = 696$$

$$53 \times 6 = 6 \times 50 + 6 \times 3 = 300 + 18 = 318$$

You try

$$87 \times 5$$

$$73 \times 6$$

More difficult

$$28 \times 9 = 9 \times 20 + 8 \times 9 = 180 + 72 = 200 + 52 = 252$$

You try

What is the area of a triangle with the base of 59” and height of 14”?

Using complements

$$7 \times 59 = 7 \times 60 - 7 = 420 - 7$$

$$78 \times 4 = 80 \times 4 - 2 \times 4 = 320 - 8$$

Distributive law

$$3 \times 87 = 261$$

$$3 \times 87 = 3 \times 80 + 3 \times 7 = 240 + 21 = 261$$

$$7 \times 324 = 7 \times 300 + 7 \times 20 + 7 \times 4 = 2100 + 140 + 28 = 2240 + 28 + 2268$$

Three by one problems

$$7 \times 300 = 2100$$

$$7 \times 320 = 7 \times 300 + 7 \times 20 = 2100 + 140 = 2240$$

$$7 \times 324 = 7 \times 300 + 7 \times 20 + 7 \times 4 = 2100 + 140 + 28 = 2240 + 28 + 2268$$

You try

How many days occur in a 6 year period?

When zero is in the middle, you can start saying your answer while calculating the rest

$$402 \times 9 = 36 \text{ hundred } 18$$

Same when there is one in the middle

$$812 \times 3 = 24 \text{ hundred } 36$$

When there is a 5 multiplied by an even number

$$521 \times 8 = 4 \text{ thousand } 21 \times 8 = 4000 + 160 + 1 \times 8 = 4168$$

$$925 \times 8 = 8 \times 900 + 25 \times 8 = 7200 + 200 = 7400$$

You try

$$789 \times 7 =$$

Another way

$$789 \times 7 = 800 \times 7 - 11 \times 7 =$$

2. Two by twos

Factorization

$$23 \times 16 = 23 \times 8 \times 2 = (160 + 24) \times 2 = 184 \times 2 = 368$$

Associative law of multiplication

$$23 \times 16 = 23 \times 4 \times 4 = 92 \times 4 = 360 + 8 = 368$$

You try

Suppose your phone service cost you \$59 per month, how much will you pay for one year?

Suppose a swimming pool is 14 ft by 14 ft by 7 ft. How many cubic feet is the volume of the pool?

Secrets of Mental Math – Division

1. When does one number divide evenly into another?

A number is divisible by

2 if and only if it ends in 0, 2, 4, 6, or 8.

5 if and only if it ends in 0 or 5

10 if and only if it ends in 0

4 if and only if its last two digits create a number divisible by 4

Is 276485328 divisible by 4?

8 if and only if its last three digits create a number divisible by 8

Is 123456 divisible by 8?

Is 3145926 divisible by 8? (hint: check divisibility by 4)

3 if and only if its digits add up to a number divisible by 3

Is 12345 divisible by 3?

9 if and only if its digits add up to a number divisible by 9

Is 12345 divisible by 9?

Exercise: show that the rules of divisibility by 3 and 9 are correct (hint: start with 9).

11 if and only if you alternately subtract and add the digits and you end up with zero or a multiple of 11.

Is 843689 divisible by 11?

6 if and only if it is divisible by 2 and 3

12 if and only if it is divisible by 3 and 4

7: take off the last digit of the number, double it, subtract from the rest of the number, if what you get is divisible by 7 then the number is divisible by 7.

Is 112 divisible by 7?

Is 2345 divisible by 7?

Can your birthday ever be on the same day two years in a row?

7 again: "Create a zero, kill a zero rule"

1234 create a zero by adding or subtracting a multiple of 7.

$1234 - 14 = 1220$ kill a zero

$122 + 28 = 150$

15 not divisible by 7, so 1234 is not divisible by 7.

This rule can be used for any odd number except for 5.

Is 2001 divisible by 23?

2. Let's look at one digit division now.

$79 : 3$, think of 3 goes 2 times into 7 so subtract 3×20 from 79 to get an easier problem of $20 + 19 : 3 = 20 + 6 + 1/3 = 26$ and $1/3$.

Covert from Fahrenheit to Celsius: 79F

$$76 - 32 = 44$$

$$44 \times 5 = 220$$

$$220/9 = ?$$

9 goes 2 times into 22 so it goes 20 times into 220, $220 - 180 = 40$, now $40/9 = 4$ and $4/9$, so the answer is 20 and 4 and $4/9$ or 24 and $4/9$.

Try 100F.

Try $777/4 = ?$

$1234/5 = ?$ Here is trick: double both numbers, then divide.

Practice:

$353/14 = ?$

$500/73 = ?$

This problem can be solved easier using "Overshooting".

$770/79 = ?$

Practice division by 2 digit numbers:

$2001/23 = ?$

$2012/24 = ?$ (hint: reduce by first dividing both numbers by 4)

$314/16 = ?$

$695/25 = ?$ (hint: when numbers end in 5 doubling may simplify the problem)

3. Decimals.

$1/2 = 0.5$

$1/3 = 0.333...$

$2/3 = 0.666...$

$1/4 = 0.25$

$3/4 = 0.75$

$1/5 = 0.2$

$2/5 = 0.4$

$3/5 = 0.6$

$4/5 = 0.8$

$1/6 = 0.1666...$

$5/6 = 0.8333...$

$1/8 = 0.125$

$3/8 = 0.375$

$5/8 = 0.625$

$7/8 = 0.875$

$1/9 = 0.111...$

$2/9 = 0.222...$

$4/9 = 0.444...$

etc.

$1/11 = 0.0909...$

$2/11 = 0.1818...$

$3/11 = 0.2727...$

etc.

$1/7 = 0.142857142857142857...$

Look at 1, 4, 2, 8, 5, 7, you can create all other fractions. Note $1/7$ is about 0.14 so

$2/7$ is about 0.28, so $2/7 = 0.285714$ repeated

$3/7$ is about 0.42 so $3/7 = 0.428571$ repeated

Try:

$4/7 =$

$5/7 =$

$6/7 =$

Now try

$3.0/16$ as a decimal

$5.0/14$ as a decimal

4. Exercises:

Determine which numbers between 2 and 12 divide into each number below:

4410

7062

2744

33,957

Use the “create a zero, kill a zero” method to test to test the following:

Is 4913 divisible by 17?

Is 3141 divisible by 59?

Is 355,113 divisible by 7?

Mentally do the following divisions

$97/8$

$63/4$

$159/7$

$4668/6$

$8763/5$