Probability, Polynomials, and Weird Dice

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PART I

1. Make a table listing the possible sums when you roll a pair of standard dice.

Table of sums for two 6-sided dice (addition table)

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

2. Now make a table of the frequencies of the sums, and the probability of rolling that sum with a pair of standard dice.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency											
Probability											

3. **Question:** Can we put different positive numerical labels on the dice, and still get the same frequencies (or probabilities) of the sums?

This question was first posed by Colonel George Sicherman of Buffalo, NY in 1977, in a letter to Martin Gardner. Martin Gardner wrote an article about this in *Scientific American* in 1978.

PART II

Before we tackle this question, let's try an easier case, dice with 4 sides.

1. Make a table listing the sums when you roll a pair of tetrahedral (4-sided) dice.

Table of sums for two 4-sided dice (addition table)

+	1	2	3	4
1				
2				
3				
4				

2. Now make a table of the frequencies of the sums, and the probability of rolling that sum with a pair of 4-sided dice.

Sum	2	3	4	5	6	7	8
frequency							
Probability							

3. **Question:** Can we put different numerical labels on the dice, and still get the same frequencies (or probabilities) of the sums?

PART III

Let **A** and **B** denote two standard 4-sided dice.
Let **C** and **D** denote other labelings of 4-sided dice.

Standard Labelling:

A	1	2	3	4
B	1	2	3	4

Trial labeling

C				
D				

Trial labeling

C				
D				

Trial labeling

C				
D				

Trial labeling

C				
D				

Trial labeling

C				
D				

PART IV

Let A and B denote two standard 6-sided dice.
Let C and D denote other labellings of 6-sided dice.

Standard Labelling:

A	1	2	3	4	5	6
B	1	2	3	4	5	6

Trial labeling

C						
D						

Trial labeling

C						
D						

Trial labeling

C						
D						

Trial labeling

C						
D						

Trial labeling

C						
D						

PART V : What does this have to do with polynomials?

1. Turn the labels on dice into polynomials, where each label represents an exponent of x.

Die A = (1,2,3,4,5,6) $\rightarrow$ $A(x) = x^1 + x^2 + x^3 + x^4 + x^5 + x^6$

Die B = (1,2,3,4,5,6) $\rightarrow$ $B(x) = x^1 + x^2 + x^3 + x^4 + x^5 + x^6$

2. Now multiply A(x)*B(x). *(You can go to Wolframalpha.com to multiply these.)*

3. What do you notice?

PART VI

Let's try this with 4-sided dice, and try to analyze the situation.

1. Turn the labels on dice into polynomials, where each label represents an exponent of x.

Die A = (1,2,3,4) $\rightarrow$ $A(x) = x^1 + x^2 + x^3 + x^4$

Die B = (1,2,3,4) $\rightarrow$ $B(x) = x^1 + x^2 + x^3 + x^4$

2. Factor A(x).

3. Now multiply A(x)*B(x), keeping it factored.

4. By choosing different factors as C(x) and D(x), we can get different dice labels.