**Math Circle** # Modular Arithmetic **Jan 21, 2013**

**1a.** Write the multiplication table for multiplication modulo 11 (on an 11-hour clock).

| $\times$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

Hints:

- Start by filling in the diagonals. For example $5 \times 5 = 25 \equiv 3 \pmod{11}$, since $25 = 2(11) + 3$.
- Count right from each diagonal square, counting by the number of the row. For example, from $4 \times 4 = 5$, fill in $4 \times 5 = 5 + 4 = 9$, then $4 \times 6 = 9 + 4 = 13 \equiv 2$, etc.
- Finally, fill in the columns below the diagonal from the corresponding rows by the commutative law: $5 \times 4 \equiv 9$, $6 \times 4 = \equiv 2$, etc.

**b.** Recall that the inverse $a^{-1}$ means the mod-11 number which cancels $a$ so that $a \times a^{-1} \equiv 1$. For example, $2^{-1} = 6$, since $2 \times 6 = 12 \equiv 1 \pmod{11}$. Find the inverse of every number $0, 1, 2, \ldots, 10$, if there is one.

**2.** Having the inverse $a^{-1}$ allows us to divide by $a$, intepreting $b \div a$ as $b \times a^{-1}$.
Find all solutions (if any) for $x$ in the following equations:

**a.** $5x + 7 \equiv 3 \pmod{11}$

**b.** $x^2 \equiv 4$

**c.** $x^2 \equiv 3$

**3a.** If a whole number $n$ is divisible by 11, then $n \equiv$ what number mod 11?

**b.** Try reducing mod 11 to see if 11 evenly divides this number: $243 = 2 \times 10^2 + 4 \times 10 + 3$.

**c.** Can you change one digit of 243 so that 11 does divide it?

**d.** Find a simple rule with the digits of a number (similar to the Rule of 3) to decide whether 11 divides evenly or not.