CHOMP THE GRAPH
A Mathematical Game of Strategy

Definitions:

- Graph – a finite set of vertices which are connected by edges
- Simple graph – a graph that is undirected with no loops or multi-edges
    - A pair of vertices has at most one edge between them
    - Edges have no orientation or direction, all edges identical to each other
- Degree of a vertex – the number of edges that connect to it
- Tree - a connected graph with no cycles
    - Number of edges (*e*) is equal to one less than the number of vertices (*v*): $e = v - 1$
- Leaf – a vertex of degree one in a tree
- Linear graph – a tree with every vertex having degree one or two
- Forest – the union of one or more trees
- Cycle – a graph with the vertices connected in a closed chain
    - There is a path that visits all vertices and does not repeat edges
    - $e = v$
- Bipartitie/utility graph – graph with the vertex set being the union of two disjoint sets, *W* and *X*
    - Vertices in *W* connect by edges to vertices in *X*, but there are no edges within *W* or *X*
- Complete graph – a graph with each pair of vertices having an edge connecting them
    - $v+e = (v)(v+1)/2$
- Winning strategy –moves that force a win no matter what moves the opponent makes
- Losing position – every move you make places your opponent in a winning position

Method of play for Chomp the Graph:

- Chomp the Graph is a game for two players
- Players take turns removing vertices and edges from a graph
- Use an ipad app ("Doodle Buddy") which allows you to draw a graph and erase as you play.
- Or use pen to draw graph and use highlighter to mark edges and vertices as they are "removed"
- **To play, first draw a simple graph**
- **On your turn you may remove one vertex and all of its incident (connected) edges** (you MUST take all the edges which are connected to a vertex)
- **Or you may remove one edge**
- The winner of the game is the one who removes the last vertex of the graph

Challenge:

- Play a few games on linear graphs which have 2 to 5 vertices
- See if you can determine a winning strategy
- Play a few games with tree graphs of other shapes, does the winning strategy change?
- Next play Chomp the Graph with cycle graphs, what is the winning strategy for these?
- If time permits, play a few games with forests, bipartite and complete graphs to determine the winning strategy for each of these graphs.