

Jim and Nim

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New York Math Circle

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Outline

1. Games



Outline

1. Games
2. Nim



Outline

1. Games
2. Nim
3. Strategies



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1. Games
2. Nim
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4. Jim



Outline

1. Games
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4. Jim
5. The Winning Strategy for Nim



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6. *Addition of Games*



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6. *Addition of Games*
7. *Equivalence of Games*



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1. Games
2. Nim
3. Strategies
4. Jim
5. The Winning Strategy for Nim
6. *Addition of Games*
7. *Equivalence of Games*
8. *The Sprague-Grundy Theorem*



But First ...



1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31



$$13 =$$



$$13 = 8 + 4 + 1$$



$$13 = 8 + 4 + 1 = 01101_2$$



$$13 = 8 + 4 + 1 = 01101_2$$

$$\frac{0}{16} \quad \frac{1}{8} \quad \frac{1}{4} \quad \frac{0}{2} \quad \frac{1}{1}$$



$$13 = 8 + 4 + 1 = 01101_2$$

$$\begin{array}{ccccc} \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \hline 16 & 8 & 4 & 2 & 1 \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$



$$13 = 8 + 4 + 1 = 01101_2$$

$$\begin{array}{ccccc} \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{1} \\ \frac{\quad}{16} & \frac{\quad}{8} & \frac{\quad}{4} & \frac{\quad}{2} & \frac{\quad}{1} \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

0 <u>1</u> 000	0 <u>1</u> 001	0 <u>1</u> 010	0 <u>1</u> 011
0 <u>1</u> 100	0 <u>1</u> 101	0 <u>1</u> 110	0 <u>1</u> 111
1 <u>1</u> 000	1 <u>1</u> 001	1 <u>1</u> 010	1 <u>1</u> 011
1 <u>1</u> 100	1 <u>1</u> 101	1 <u>1</u> 110	1 <u>1</u> 111



Express in Binary

$$6 =$$



Express in Binary

$$6 = 4 +$$



Express in Binary

$$6 = 4 + 2$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 =$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 +$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1 = \boxed{1} \cdot 2^2 + \boxed{0} \cdot 2^1 + \boxed{1} \cdot 2^0$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1 = \boxed{1} \cdot 2^2 + \boxed{0} \cdot 2^1 + \boxed{1} \cdot 2^0 = 101_2$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1 = \boxed{1} \cdot 2^2 + \boxed{0} \cdot 2^1 + \boxed{1} \cdot 2^0 = 101_2$$

$$3 =$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1 = \boxed{1} \cdot 2^2 + \boxed{0} \cdot 2^1 + \boxed{1} \cdot 2^0 = 101_2$$

$$3 = 2 +$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1 = \boxed{1} \cdot 2^2 + \boxed{0} \cdot 2^1 + \boxed{1} \cdot 2^0 = 101_2$$

$$3 = 2 + 1$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1 = \boxed{1} \cdot 2^2 + \boxed{0} \cdot 2^1 + \boxed{1} \cdot 2^0 = 101_2$$

$$3 = 2 + 1 = \boxed{0} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{1} \cdot 2^0$$



Express in Binary

$$6 = 4 + 2 = \boxed{1} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{0} \cdot 2^0 = 110_2$$

$$5 = 4 + 1 = \boxed{1} \cdot 2^2 + \boxed{0} \cdot 2^1 + \boxed{1} \cdot 2^0 = 101_2$$

$$3 = 2 + 1 = \boxed{0} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{1} \cdot 2^0 = 011_2$$



Which is Bigger?



Which is Bigger?

Base 10:



Which is Bigger?

Base 10: 1894 or 2011



Which is Bigger?

Base 10: 1894 or 2011

1920 or 1993



Which is Bigger?

Base 10: 1894 or 2011

1920 or 1993

Base 2:



Which is Bigger?

Base 10: 1894 or 2011

1920 or 1993

Base 2: 00101 or 10101



Which is Bigger?

Base 10: 1894 or 2011 1920 or 1993

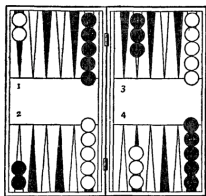
Base 2: 00101 or 10101 10101 or 10011

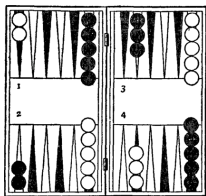


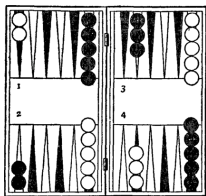
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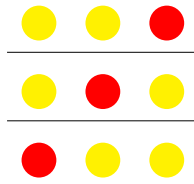
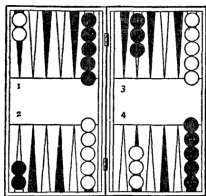












Nim



Nim Challenges

Who has the winning strategy?



Nim Challenges



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Who has the winning strategy?

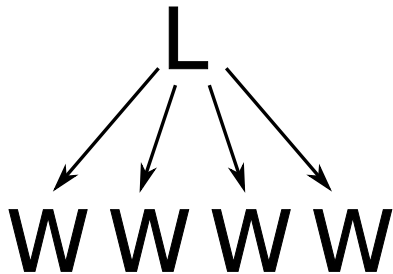


Nim Challenges

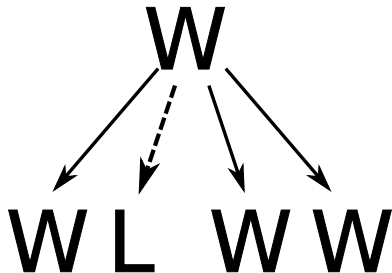
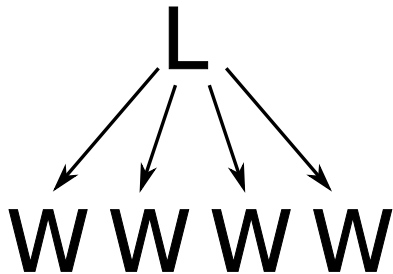
Which two pile Nim games are L positions?



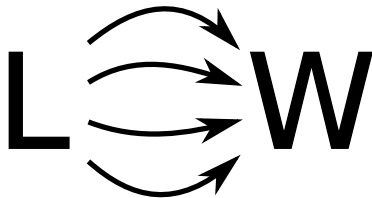
W and L games



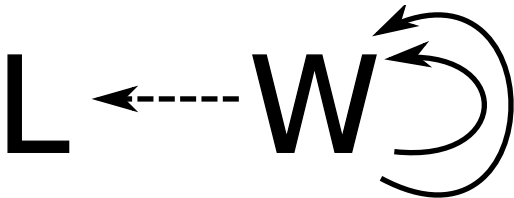
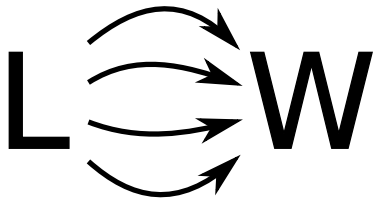
W and L games



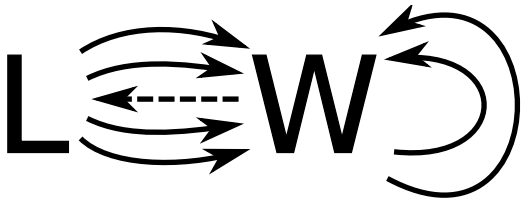
W and L games



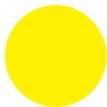
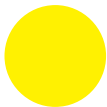
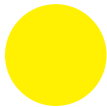
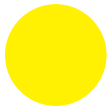
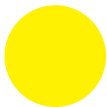
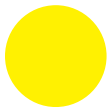
W and L games



W and L games



Jim



Jim Challenges



Who has the winning strategy?



Jim Challenges



Who has the winning strategy?



Jim Challenges



Who has the winning strategy?



Jim Challenges



Who has the winning strategy?



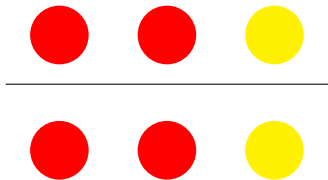
Jim Challenges



Who has the winning strategy?



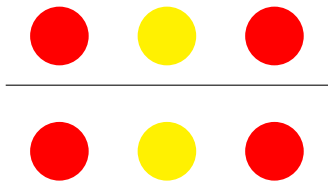
Jim Challenges



Who has the winning strategy?



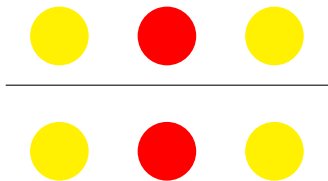
Jim Challenges



Who has the winning strategy?



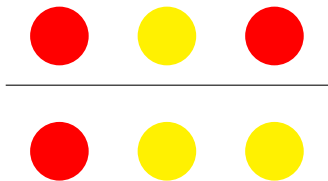
Jim Challenges



Who has the winning strategy?



Jim Challenges



Who has the winning strategy?

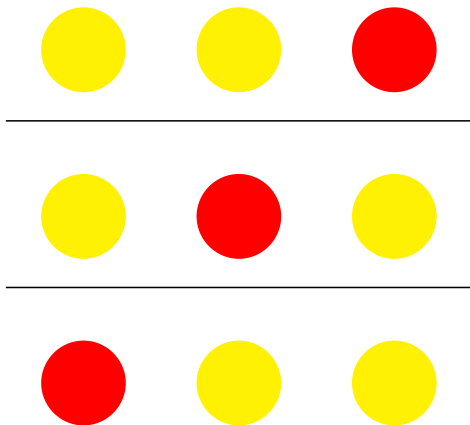


Jim Challenges

Which two row Jim games are L positions?



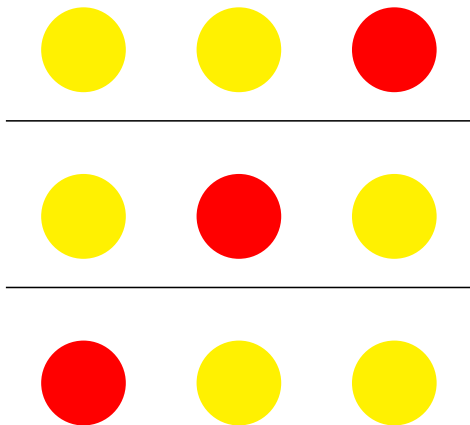
3-Row Jim



Show this is an L position.



3-Row Jim



Goal: Describe all 3-Row Jim L positions.

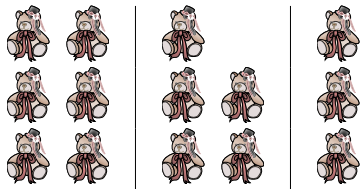


Jim and Nim

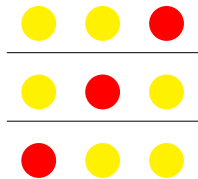
Can a Strategy for Jim help solve Nim?



Jim and Nim



6 | 5 | 3



1	1	0
<hr/>		
1	0	1
<hr/>		
0	1	1

Can a Strategy for Jim help solve Nim?



Thank You!

Nim and Jim

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Some Games Collected by David Hankin

There are 5 checkers on a table. A move consists of taking one or two checkers from the table. The winner is the one who takes the last checker.



Some Games Collected by David Hankin

There are 100 checkers on a table. A move consists of taking m checkers from the table, where m is a positive integer power of 2. The winner is the one who takes the last checker. Find the set L of losing positions.



Some Games Collected by David Hankin

There are 100 checkers on a table. A move consists of taking m checkers from the table, where m is a prime or $m = 1$. The winner is the one who takes the last checker. Find the set L of losing positions.



Some Games Collected by David Hankin

There are 100 checkers on a table. A move consists of taking 1, 3, or 8 checkers from the table. The winner is the one who takes the last checker. Find the set L of losing positions.



Some Games Collected by David Hankin

There are two piles of checkers on a table. A move consists of taking any number of checkers from one pile or the same number of checkers from each. The winner is the one who takes the last checker. Find the set L of losing positions.



Some Games Collected by David Hankin

Given an initial integer $n_0 > 1$, two players, A and B , choose integers n_1, n_2, n_3, \dots alternately according to the following rules. Knowing n_{2k} , A chooses any integer n_{2k+1} such that $n_{2k} \leq n_{2k+1} \leq n_{2k}^2$. Knowing n_{2k+1} , B chooses any integer n_{2k+2} such that n_{2k+1}/n_{2k+2} is a positive power of a prime. Player A wins by choosing the number 1990, player B wins by choosing the number 1. For which n_0 does

A have a winning strategy,

B have a winning strategy,

neither player have a winning strategy?

