

Pennies, Piles, and Games

In all the games described below there are two players. The problem is to decide which one of the two players has a winning strategy, and to describe the strategy. One very important part of the strategy is to decide whether you want to go **first** or **second**.

We recommend that you start with the first game, play it several times and then answer all the questions. Then go to the next game and do the same. The games become more challenging (and more interesting!).

In each game, the **winner** is the person who gets the **last** penny. It doesn't matter how many you get!

1. Begin with one pile of pennies on the table. On your turn, you take **one** penny.
 - (a) If the pile has 3 pennies, do you want to go first or second? After you've made that decision, what strategy do you use?
 - (b) What if there are 4 pennies?
 - (c) What if there are 5 pennies?
 - (d) What about 10 pennies? 50 pennies?
 - (e) How do you decide whether to go first or second, and what to do after that, once you know how many pennies are in the pile?
2. Begin with one pile of pennies on the table. On your turn, you take **one, two, three, or four** pennies.
Now repeat the questions from the previous problem.
3. Begin with one pile of pennies on the table. On your turn, you can take any number of pennies as long as it is **less than half** of the whole pile, although even if there are only one or two pennies left, you are still allowed to take one. (There have to be at least 5 for you to be allowed to take two!)
Now repeat the same questions (above), although maybe jumping from 5 to 10 to 50 will be too challenging now!
4. Begin with **two** piles of pennies on the table. On your turn, you take **as many pennies as you want** from **one** of the two piles (including taking the whole pile, if you like).
 - (a) If your piles have 7 and 10 pennies, what should you do?
 - (b) When you're faced with a hard problem, try an easier one: 1 penny in each pile, perhaps?
 - (c) What about 2 and 1 for your two piles? 2 and 2? 50 and 1?
 - (d) How do you decide whether to go first or second, and what to do after that, once you know how many pennies are in each pile?