The Magic Birthday Trick

I will guess the day of the month that you were born!

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

21-Nim

Here's a great warm-up game that is similar to Nim. There is one pile of 21 counters. On your turn, you can take 1, 2 or 3 counters. You win if you take the last counter. In a *losing* position, the next player will lose if her opponent plays correctly. Challenge: find and box the set of *losing positions*!

$\boxed{0}$ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

The Game of Nim

This is the real game of Nim, whose winning strategy was first described and proved in a math research paper published in 1901. The game starts with several piles of counters. Players alternate moves: on your turn, select a pile and remove at least one counter from that pile. You win if you take the last counter from the last pile. That is, the last player with a legal move wins. If you know the set of *losing positions*, then you know the winning strategy!

Nim Challenges

Which of the following are *losing positions*? Notation: 3 denotes a pile of of 3 counters, 3 + 2 denotes two piles, one with 3 counters and one with 2 counters. The 0 is the empty Nim game with zero counters. Since the empty game is a losing position, I've boxed that to get you started.

$$\begin{array}{ccccccccccccccccc}
5 & & 4+1 & & 3+2 & & 2+3 & & 1+4 & & 5 \\
& 4 & & 3+1 & & 2+2 & & 1+3 & & 4 & \\
& & 3 & & 2+1 & & 1+2 & & 3 & & \\
& & & 2 & & 1+1 & & 2 & & & \\
& & & & 1 & & 1 & & & & \\
& & & & & \boxed{0} & & & & &
\end{array}$$

1. Make a conjecture about which two pile Nim games are *losing positions*. Prove it!
2. Explain why 3 + 5 + 6 (three Nim piles, with 3, 5 and 6 counters) is a *losing position*.

The Game of Jim (Japheth's Nim)

Similar to Nim, Jim was invented by Bard math professor Japheth Wood. A Jim game starts with several rows of solid tokens • and empty tokens ∘. Players alternate moves: select a row, and change one or more tokens (solid to empty or empty to solid). **RULE:** The first token-change from the left *must* be solid to empty (but does not need to be the leftmost solid token). The last player with a legal move wins. Equivalently, if you only see empty tokens, then you've just lost!

Jim Challenges

Which of the following Jim games are *losing positions*?

1. One row Jim: ∘ ∘ ∘ (all empty tokens). One row Jim: • ∘ •.
2. Two row Jim: • ∘ • | • ∘ •. Two row Jim: ∘ ∘ • | • ∘ •.
3. Make a conjecture about which one row and two row Jim games are *losing positions*.
4. Explain why this three row Jim game is a *losing position*: ∘ • • | • ∘ • | • • ∘.

Nim and Jim

1. What is the *maximum* number of moves for a one-pile game of Nim with 5 tokens?
2. Can you find a one-row Jim game that has the same maximum number of moves?
3. Make a conjecture about the connection between Nim and Jim games.

Math Teachers' Circles

Math Teachers' Circles are fantastic professional development opportunities for teachers seeking to deepen their math content knowledge, develop their problem-solving skills, work directly with mathematicians and become part of a growing community of math teachers.

The **New York Math Circle** has activities for teachers throughout the year in NYC, and an immersion workshop during the last week of July at Bard College. Teachers from all over the country are welcome. URL: `http://nymathcircle.org/teachers`.

The **Math Teachers' Circle Network** maintains a list of math teachers' circles around the country. Get involved! URL: `http://www.mathteacherscircle.org/workshops.html`.

For Further Reading

Check out these important works on full-information two-player games!

Charles L. Bouton, *Nim, A Game with a Complete Mathematical Theory*, The Annals of Mathematics Vol. 3, No. 1/4 (1901 to 1902), 35–39.

Elwyn R. Berlekamp John H. Conway and Richard K. Guy, *Winning Ways for your mathematical plays,* Academic Press, London, 1982.