A Penny for Your Thoughts ...

1. Let a collaction of pennies lie flat on a table.
   (a) What is the fewest pennies that a given penny can touch? What is the most? Why?
   (b) Is it possible for 25 pennies to be arranged, so that each penny touches exactly 3 other pennies? Is it possible to do this with 24 pennies?
   (c) For which numbers n, is it possible to arrange n pennies so that each penny touches exactly 3 other pennies?
   (d) What happens if each penny must touch exactly 4 other pennies?
   (e) What happens if each penny must touch exactly k other pennies?
2. Now consider balls in 3-space. Can you find a configuration in which 12 unit balls touch some other unit ball? (A unit ball is one of radius 1.)
3. For which numbers n, is it possible to arrange n unit balls so that each ball touches exactly 3 other unit balls?
4. For which numbers n, is it possible to arrange n unit balls so that each ball touches exactly 4 other balls?
5. What is the most unit balls that can touch a given unit ball? This is tricky. The answer is known in dimensions 3, 4, 8 and 24, but it is unknown in dimension 6.

It is possible to make a graph from a collection of balls in space. Let each ball be a vertex and connect two balls with an edge exactly when the balls touch. This graph is called the contact graph of the configuration. It is NP hard to decide if a given graph is a contact graph. This is related to coding theory, as well. An error detecting code is a subset of the binary representations of the numbers 0 through $2^n - 1$. It must have the property that if any one bit in a code word is changed, the result will no longer be a code word. For example the parity bit code on the numbers $\{0, 1, 2, \ldots, 8 - 1\}$ is $\{000, 011, 101, 111\}$. Notice that these all have even bit sums. Thus, changing one bit, say $011 \mapsto 001$ results in a word with the wrong bit sum, so we know that an error occurred in transmission.

Draw the lattice of all integer vectors in $\mathbb{Z}^3$ having coordinates congruent modulo 2 two a code word. This is the face centered cubic lattice. It is natural to consider the subspace consisting of integer vectors with zero component sum. Draw this planar lattice. Put the largest penny possible at the center of each lattice point in this planar lattice. How many other pennies touch each given penny? Put the largest balls possible around each lattice point in the face centered cubic lattice. How many other balls touch each given ball?