Name that Polynomial

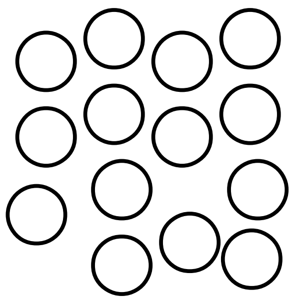
Problem
I'm thinking of a polynomial in x with whole number coefficients. When x = 1, the value of the polynomial is 22. When x = 25, the value of the polynomial is 33,334. What is the polynomial?

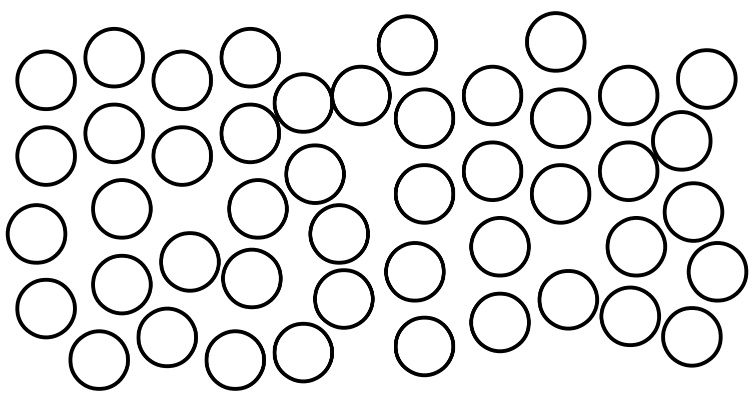
The Arithmetic of Dinner Rolls

Baked goods and eggs are often sold by the dozen, which is a term from the following set of units:

12 objects	=	1 dozen
12 dozen	=	1 gross
12 gross	=	1 great gross

How many objects are there in each of the two sets? Express this number in objects, dozens, gross, and great gross.





Suppose you had containers for single objects; dozens; gross; and great gross. Being ecologically conscious, you'd like to use as few containers as possible. What containers, and how many of each, would you need to package the following amounts? For example, 18 rolls *could* be packaged into 18 individual containers, but it might be possible to use fewer containers to package them.

18 rolls

3 dozen 25 rolls

2 gross 10 dozen 24 rolls

More Arithmetic

All of your answers should be expressed in dozens, gross, and great gross as necessary.

Suppose you have one bag containing 5 dozen 7 rolls, another bag containing 3 dozen 6 rolls, and a third bag containing 6 dozen 8 rolls. How many rolls do you have all together?

You have 7 bags, each containing 1 gross 4 dozen 3 dinner rolls. How many dinner rolls do you have all together?

.

You have 1 dozen bags, each containing 1 gross 4 dozen 3 dinner rolls. How many dinner rolls do you have all together?

You have 1 dozen and 5 bags, each containing 3 dozen and 7 dinner rolls. How many dinner rolls do you have all together?

On a Roll

You send 3 gross 2 dozen dinner rolls for a party. If 2 gross 8 dozen 5 dinner rolls are eaten, how many remain? Express your answer using dozens, gross, and great gross as necessary

You need to split a bag containing 5 dozen and 7 dinner rolls into 3 bags containing the same number in each. How many rolls are in each bag? How many rolls do you have left over?

You have a bag containing 9 dozen and 7 dinner rolls. How many bags containing 2 dozen 5 dinner rolls apiece can you make from it? How many rolls do you have left over?

You have a bag containing 2 gross 3 dozen and 8 dinner rolls. How many bags containing 1 dozen 5 dinner rolls apiece can you make from it? How many rolls do you have left over? (Express your answers in dozens and ones)

Breaking Bread

You're ordering the bread for the Endekaphile Banquet. Each table has 11 persons, and each person should have one dinner roll.   Currently, you don't know how many tables there will be.

Suppose there's only one table. How many rolls do you need to order? (Remember that your answer has to be expressed using units, dozens, and gross)

Suppose there are two tables. How many rolls do you need to order?

Suppose there are three tables. How many rolls do you need to order?

Suppose there are twelve tables. How many rolls do you need to order?

Suppose there are fifteen tables. How many rolls you need to order?

All of these numbers should be multiples of 11. What do you notice about them?

Suppose the bakery delivers an order of 5 gross 8 dozen 3 dinner rolls. Could this be an order for the Endekaphile Banquet? Why or why not?

Algebra is Generalized Arithmetic

An expression like "5 gross 7 dozen and 3 dinner rolls," is known as the *dispositional form* of a number. Because the units differ by factors of 12 (12 ones is 1 dozen; 12 dozen is 1 gross; 12 gross is 1 great gross), we say that the system is *base twelve*. We can take one more step and drop the unit names to write the *base twelve positional form*: 573_{twelve}, where we've indicated our base (twelve) by spelling it out and subscripting it. The alert reader will note that if we *just* drop the unit names, we confront the following problem:

Dispositional Form	11	11 dozen	1 dozen and 1	1 gross and 1
Positional Form	11_{twelve}	11_{twelve}	11_{twelve}	11_{twelve}

We leave the resolution of this problem as an exercise!

The numeration system we use is a base ten positional system. In dispositional form, we might have a number like 3 hundred 2 tens 8, which becomes the positional 328_{ten}. Actually, since base ten positional is our "normal" form of writing numbers, we usually omit the subscript and just write 328.

At some point, we learned standard algorithms for the basic operations of arithmetic. But today, when a computing device is no further than your cell phone, the value of the standard algorithms is not obvious. The real importance of arithmetic is *not* the ability to multiply 37 × 28 without a calculator. Rather, arithmetic is important because algebra *is* arithmetic, generalized.

For example, every arithmetic algorithm is a method of keeping track of the number of ones, tens, and hundreds. Thus the addition 5 dozen 7 + 3 dozen 6 + 6 dozen 8, the addition $(5x + 7) + (3x + 6) + (6x + 8)$, and the addition 57 + 36 + 68, are essentially the same problem. Likewise, finding 7 × 143 is not essentially different from finding the number of dinner rolls in 7 bags each containing 1 gross 4 dozen 3 dinner rolls, and we can apply the same process to finding $7(x^2 + 4x + 3)$.

Now consider an expression like $5x^2 + 3x + 7$. If we let $x = 10$, then $5x^2 + 3x + 7$ evaluates to 537, where the digits are of the number (expressed in base ten) are the same as the coefficients of the polynomial. On the other hand, if we let $x = 12$, the value of $5x^2 + 3x + 7$ is 763---which is the same as 5 gross 3 dozen 7, or 537_{twelve}. Again, the digits of the number (expressed in base twelve) are the same as the coefficients of the polynomial. This means that in general, a polynomial can be treated as a multidigit number in base x. Thus if you know the value of a polynomial for a given value of x, you can recover the coefficients.

Why did we need the first question? Consider a polynomial like $10x^3 + 4x^2 + 8x + 11$. If we let $x = 10$, this evaluates to 10,491. But our polynomial is not $1x^4 + 0x^3 + 4x^2 + 9x + 1$: the problem is that two of the coefficients, the 10 and the 11, exceed 10. In this case, since the largest coefficient is 11, we need to take an x-value greater than 11 to find a usable value. The first question gave us the answer to the question "What is the sum of the coefficients?", which put an upper bound on the largest coefficient. In this case, if we knew the largest coefficient was 11, then we could evaluate the polynomial at $x = 12$: 17,963. We could then convert this to 10 great gross 4 gross 8 dozen 11 ones, which would give us the coefficients of our polynomial.

And our polynomial, which was 33,334 when $x = 25$? By the preceding logic, the base-twenty-five expression of 33,334 will give the coefficients of the polynomial. We leave finding the expression as an exercise for the reader.