Problems kids love to solve

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Short intro

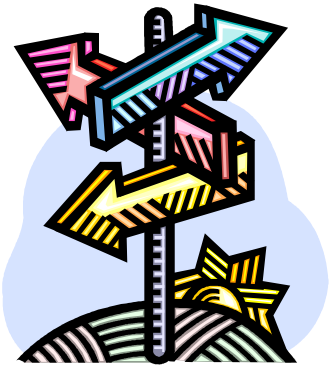
- Julia Brodsky, mom of 3 kids, engineer
- Leading math circle for elementary school kids
- Supported by MSRI mini-grant
- www.artofinquiry.net

Math circle – moving towards an “ideal world”

- “ The only fun time in school is lunch”
  - What makes school boring?
- School focuses on what “needs” to be taught, not what is interesting to a child
  - Children conditioned to solving only easy, one-step problems are having great difficulties in problem solving later on
  - Young children are motivated by complex problems with multiple solutions

Freedom of choice

- You are able to choose your problems
- What are your criteria?
- Skill relevance is not a good one for elementary school students
  - You may safely assume that they have no math skills.
- Let's concentrate on "interesting" problems that make students think



Problem selection

- Since there is no need to cover specific topics, we can concentrate on problems themselves
- We are looking for interesting problems that keep the child engaged and motivated
- Motivation comes from the nature of the problem
  - No “carrots” (good grade, praise, etc)
  - No dependence on fake entertainment

Interest vs skill

- Teaching “from simple to complex” is a proven method to teach a skill
- But does this approach create interest?
- Most of the time, it **does not**.

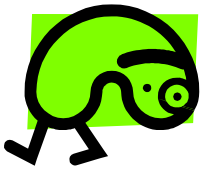
- “If you show me how to put this puzzle together, it is not interesting to play with it anymore”

  - a 5-year old

“Mystery” component

- *Observation:* Young kids love solving problems they have no idea know how to solve
- Mystery is essential…
- Teaching from simple to complex frequently takes “mystery” out of the picture
- How can we keep the mystery “alive”?

Teaching “backwards”



- What if we start from suggesting a complex problem, and building “top-down”?
- *Observation*: Kids love to attempt solving <u>complex</u> problems with <u>simple</u> statements

  a) Why does water reflect light, and grass does not?

  b) For a given perimeter, which figure has a larger area – square or rectangle? Why?
- *Observation*: Giving hints along the way works much better than “straightforward” teaching (see next slide)

Alcuin's "grain problem"

- **A head of household has 20 servants. He ordered them to be given 20 sacks of grain as follows: men will receive 3 sacks, women 2 sacks, and the children, half of a sack. How many men, women and children are there, if there is at least one of each?**
- *Individual work; several partial solutions are found by trial and error.*
- *Teacher: "Are there any limitations on the number of men, women and children?"*
  - *"You can not have more than 6 men"*
  - *"There should be an even number of children"*
  - *"You can not have more than 8 women" , etc.*
  - *"There are several solutions that are fine for 20 sacks; but if you take a limitation of 20 people, there is only one solution"*
- *(Solution: 14 kids, 5 women, 1 man)*

Complex problems vs knowledge

- It is often assumed that solving complex problems always requires a lot of knowledge
- However, we need to distinguish between the knowledge needed to understand the problem and the knowledge needed to solve it
  - Many complex problems can be stated in such simple terms that they can be grasped even by a young child
  - These problems can create significant interest in a child, leading her to explore on her own

Example: Gnomes vs Dragon

- Ten gnomes are caught by a dragon. The dragon lines the gnomes up as shown. The dragon has a large bag full of black and white hats. The dragon randomly reaches into his bag and places a hat on each of the gnomes. The dragon asks the gnome, "What color is your hat?" If the gnome answers correctly, he lives and gets freed from the dungeon. If he does not, he dies. He continues up the line in this progression. However, before placing the hats on the gnomes, he allows the gnomes to meet as a group and discuss a strategy to save as many of the gnomes as possible. Imagine that you are one of these gnomes. What strategy would you develop? How many gnomes can you guarantee to save? (You must ONLY say "white" or "black")
- (For more advanced students, consider any number of gnomes)

Open ended problems

- Open-ended problems are always welcomed by kids
- Sometimes, they are eye-opening for teachers, too

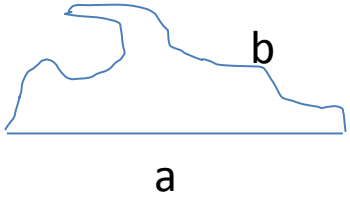
- *Example*:
  - Find a way to add all numbers from 1 to 999
- *Solution* (9-year old)
  - Let's first find S= 1+..99
    - S1= 1+..+9
    - S2= 10*S1
    - S= S2 + 9*S1
  - 1+..+999 by analogy

Exploratory problems

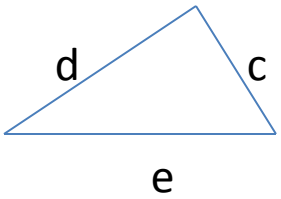
- Take a compass and a ruler. Try to construct a triangle with the sides 1, 3, and 7
  - Kids would not believe their eyes – the problem looked so easy at the first sight..

“Proofs and demonstrations without words”

- All kids agreed that b>a



- But they had trouble seeing that d+c>e



- Show that a×b=b×a

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Problems with “unexpected” solutions

roblem: A rope burns non-uniformly for exactly one hour. How do you measure 45 minutes with two such ropes?

…

- Sometimes, adults attribute problem complexity to lack of knowledge rather than their own suppressed creative ability
- There are many puzzles that may seem difficult to an adult, but are easily solved by youngsters.
  - What is the rule for separating these sets: {1,4,7} and {2,3,5,6,8,9}?
  - Where was the Declaration of Independence signed?
  - What is the difference between here and there?

Discussion suggestions

- How do you distinguish between an “interesting” and “boring” problem?
- How to keep the “mystery” alive?
- A database for “interesting” problems

“Interdisciplinary” approach

- “ Define a dog”
- “An animal with a triangular face and horizontal body”
- “Best man’s friend”
- “Animal which I am afraid of when it barks”
- “ I can give you a definition if we get rid of all other animals in the world”

- “ Define a square”
- “Face of a cube”
- “A quadrilateral such as an inscribed circle touches the midpoint of each side”
- “A quadrilateral that goes into itself after each rotation on angles, that are multiples of a right angle.”