Soma Cube

The Soma Cube was invented by Piet Hein, in the 1930's. It has seven pieces, which are all the ways 3 or 4 cubes can be joined face-to-face, so that the resulting shape is NOT rectangular.

1. Try to make a cube from the 7 pieces. (But don't try too long, look at the suggestions below for some hints.

2. Look at the pieces.
    a. How many small cubes are in each piece?
    b. How many different pieces could you make with 3 cubes? With 4 cubes? With 5 cubes? (Including the rectangular ones.)
    c. Which pieces are symmetric? Which have mirror images? (Do you know what symmetric means? If not, ask one of the helpers.)

3. Look at 3x3x3 Soma cube that you are trying to pack. How many corners (vertices) does it have?

4. Look at the pieces: how many corners can each one fill? What's the maximum number for each piece, and the minimum number?

5. Note that only ONE piece can fill fewer than it's maximum corners.

6. Note that the T piece will either fill no corners, or 2 corners. Can it fill no corners in the solution? Where must it go?

7. Here's a sub-problem that might help think about the Soma Cube
    a. Look at a checkerboard with two missing corners that are diagonally opposite each other.
    b. Can you tile the checkerboard with 31 dominos?
    c. Imagine the dominos are colored so that each domino has a black square and a white square.

d. How many black squares are on the checkerboard? How many white squares? How many on each domino? How many are they total in all the 31 dominoes?
e. Can you see why that makes this problem impossible?

8. Now imagine the big cube is colored like a 3-dimensional checkerboard. Are all the corners the same color? Edges? Face-centers? Center of the whole cube?

9. Look at each piece. If you checkerboard the pieces with two colors, which pieces have an equal number of each color, which have different numbers?

10. If the T piece has to go on an edge, where can the Y piece go? (The Y piece is the one that looks like a corner, and is one of the piecess that require three dimensions.)

11. The V piece also has limited places it can go, but it is easier to just put the rest of the pieces in (after the T and Y), and save the v for last. (The V piece is the one with 3 small cubes, bent to look like a v.)

12. Can you make a table to show where each piece could go in the cube?

13. Can you guess how many solutions there are in total for the Soma cube? Can you figure an upper-bound for the number of solutions.

14. There should be two shapes on the table that look like a cube but with 4 small cubes moved from the bottom layer to the top, one from the centers of the bottom edges, the other from the bottom corners. Can you make either of these shapes with the 7 Soma pieces? If not, can you prove it can't be done?

15. There are MANY ways to dissect a cube into pieces like the Soma cube, giving very many possible puzzles. Some are more difficult then Soma, and some are easier. Some have a single solution, while

others (like the Soma) that have many solutions. The Soma Cube actually has 240 different solutions! And, of course, you could make pieces for a 4x4x4 cube or even larger, though when you get too many pieces, it is not as interesting as a puzzle, though more interesting if you use a computer to solve it. You can also use the Soma pieces to make other shapes besides a cube.

Variations:

- There are LOTS of other cube dissections, and the cubes can be bigger than 3x3x3. Here are three to apply vertex counting and/or parity:
    - -- Coffins Quartet (should be easy, if you count corners.)
    - -- Reid's Cube (The biggest piece can only go one place. The second biggest piece has two possible positions.)
    - -- Vesala #4
    - -- Brams' Cubes number 6, 7, or 8

- You can color the cubes or apply other patterns, to limit the solutions. For instance, you could put numbers on each face, like a die, or checkerboard each piece and require the solution to be checkerboarded.
    - -- The orange and brown checkerboard cube still has many solutions. In fact, all the solutions can still be made, EXCEPT for one piece that is not symmetric, so colored one way has some of the solutions, and colored the other way has the rest.

- You can skew the pieces and the whole cube, so the pieces fit in fewer ways. The Rhombus Cube on the table is an example of this. The same arguments about corners and parity apply, but many of the possibilities are eliminated. This may, in fact, have only one solution. Can you either find more solutions, or prove there is only one?

- The Soma Cube uses only non-rectangular pieces. But you can replace either the tri-cube with a straight cube, or one of the tetra-cubes with a square tetra-cube, or both the tri-cube and one of the tetra-cubes at the same time. Most of those variations can still be solved, but not all. Which ones? You can use the "Soma Cube With Replacements" to try some of these variations.

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