Lesson Plan: Peering Through Tubes

The activities of this lesson are adapted from the following source:

Miller, Nathaniel (2010). Modern Geometry I. *Journal of Inquiry Based Learning in Mathematics,* 17.

Focus: Similar Triangles

Materials:
Peering Through Tubes worksheet
Construction paper or cardstock paper
Tape
Measuring tape
Chalkboard or Whiteboard*
Graph paper

Length of Activity: Approximately 1.5 hours

Intro/Hook:
Begin this activity by rolling a piece of paper into a cylindrical tube. As you are doing this, ask the participants:

- "Why might a person want to look through a tube like this?"

Look through your tube and ask:

- "What do you think I see through the other end?"
  Participants should decide that looking through a tube like this might help someone sharpen their focus on an object, but it doesn't magnify or otherwise enhance our vision.
- "What does my field of vision look like? What shape is it?"
  We eventually will measure the diameter of the field of vision so this is a good time to introduce the idea that what you see is a circular area.
- "What kinds of things might affect how much we see through the tube?"
  List responses on the board.
  (Possible Responses: amount of light in the area, the color of the tube, how good your vision is, whether you're wearing glasses, how big the tube is, how far away you are, etc.)

The goal is to have the students agree on the following ideas:

- If you are further away from a scene, you'll see more of it.
- If your tube is bigger in diameter, you'll see more.
- If your tube is longer, you'll see less.

If they're not getting to these ideas, prompt them by looking at a particular student through the tube and asking leading questions:

- “Now I can see all of John’s head through this tube. What if I stood 10 feet further back? And what if I stood closer to John?”
- Adjust the size of your tube by making the diameter larger and then smaller. “What do I see now? And if I make it a very narrow tube?”
- “What do you think you’d see if your tube was twice as long? Half as long?”

Task:
After the discussion, explain to the students they will be working in groups to:

Try to find a formula for the diameter of your field of vision through a tube.

This formula should depend on some of the things on the list, namely:

- Diameter of the tube
- Length of the tube
- Distance from the scene

Distribute the *Peering Through Tubes* worksheet and have each group first make a tube and then decide on a way to take measurements. The worksheet will help them organize their data.

*You can give them freedom to do the data collection any way they see fit, or you can encourage them to do it a particular way that will work for your space. For example:

- If you have ample blackboard or whiteboard space, to measure the diameter of the field of vision you might encourage one student to stand at the board and make a mark at the top and bottom of the view as seen by his partner who is standing a measured distance from the board.
- Alternatively, you could have one student look through her tube at a tape measure on the floor and read off the largest and smallest increments she can see to find the diameter of the field of vision, while her partner measures how far her eye is from the ground.

Once participants have collected some data with one tube, have them repeat the process for different tubes (see *Peering Through Tubes* worksheet).

When you feel a group has collected enough data, encourage them to start looking at their data to find patterns. You may want to have graph paper on hand so that they can graph the data from one tube on a set of axes, enabling them to compare the diameter of the field of vision to the distance from the floor (or wall) for that particular tube.

Discussion/Implications:
After sufficient data collection among groups, gather the participants for discussion. First, make sure everyone’s measurements agree with the hypotheses from the beginning:

    - If you are further away from a scene, you'll see more of it.
    - If your tube is bigger in diameter, you'll see more.
    - If your tube is longer, you'll see less.

Then ask for volunteers to describe any patterns they saw in their data, listing these ideas on the board. If a group says something along the lines of "The diameter times 3 gave us the distance from the board," make sure you also note the diameter and length of the tube(s) they used to get that pattern.

The goal is to discover that:

The ratio of the diameter to the length of the tube should (approximately) be equal to the ratio of the diameter to the distance of the view.

Extension:
Depending on time and student interest, you could:
- Lead them toward discovering a diagram of the situation involving similar triangles. This could lead to a discussion about similarity and why the ratios are equal.
- Have students calculate the percentage error for their data.
- Discuss why the data does not exactly fit this idea:
    - Where does the error come from?
    - How could we work on getting less error?
    - Could we ever measure perfectly?

Peering Through Tubes: How Much Can We See?

1. Your group should make a tube to start looking through. It can be any length and diameter, but make sure you can measure both dimensions easily. Record these dimensions in the first table below.

2. Have one person stand some distance from a wall (measure this distance!) and look through the tube. Another person should stand at the wall and find a way to measure the diameter of the viewing area the other person can see. Do this several times with different distances from the wall and enter your results in the first table below.

   Do you see any patterns?

3. Make another tube and repeat steps 1 and 2, entering your results in a different table.

4. Try graphing your data with:
   - *x*-axis: distance from wall
   - *y*-axis: diameter of the viewing

   How does this look?

5. Keep trying different tubes and different distances from the wall until you come up with a conjecture about what's going on. Then try again and check your conjecture.

Tube Length:	Tube Diameter:
Distance to Wall	Diameter of Viewing Area

Tube Length:	Tube Diameter:
Distance to Wall	Diameter of Viewing Area

Tube Length:	Tube Diameter:
Distance to Wall	Diameter of Viewing Area

1. What is your conjecture? Can you write a formula for calculating the viewing area if you are given a tube and told to stand a certain distance away from the wall?

2. How far off was your data? (You can calculate the percentage error by dividing the difference between your data and the conjectured value by the conjectured value and then multiplying by 100.)

3. Why was your data not perfect? What kinds of error were there?