Un-Holey Cubes (Conway Cube)

This type of cube dissection is believed to have first been studied by John Conway, who is also the inventor of the "stick" version. Whether he or someone else invented the 3x3x3 version (with 3 "holes" or 1x1x1 cubes) is not known, but it is reputed to have first been published in 1970 in a book called Cubics, by Jan Slothouber and William Graatsma. So the 3x3x3 version is sometimes called the Slothouber-Graatsma Cube. I've chosen to call this type of puzzle Un-Holey Cubes, because it needs a short name, and because the solution depends on figuring out where the "holes" (sometimes just the small, most odd-shaped pieces) go. If you take out the pieces with an odd number of small cubes, it would not change where the rest of the pieces go in the box.

1. Try to fit all the pieces into the cube.

2. Look at the pieces: what are their dimensions?

3. Look at the size of the box: how big is it?

4. How can pieces (or part of the pieces) fit into **one layer** of the cube?
   Is there an extra hole, that can't be filled by normal pieces?
   Think about the shapes of the pieces that are NOT 1x1x1 cubes or 1x1x3 sticks. How many small cubes would each one contribute to a layer? Must it be an even number? How many small squares are in each layer? Is it even or odd? Can you add up even numbers and get an odd number?

5. How many layers are there, from top to bottom?
   How many layers from front to back?
   How many from right to left?

6. If **every layer** must have a 1x1 hole, and all layers must have a hole, and there are only 3 1x1x1 small cubes, where **must** the holes be?

7. You should now be able to solve the 3x3x3 puzzle.

VARIATIONS

1. Try the 5x5x5 version, with 5 1x1x1 small cubes.

2. Try the 7x7x7 version with 1x1x1 small cubes.

3. With the 3x3x3 cubes, there is one without the 1x1x1 cubes, and one with the "holes" attached to the 2x2x1 pieces, and one where all the pieces are rhombic, at an angle. Which of the 4 versions is easiest? Which hardest? The one with no small cubes has fewest pieces, does that make it easier than the others?

4. Stick cubes (Conway Cubes): find the version that is a 5x5x5 box, with three 1x1x3 pieces. Try to solve it, with the knowledge you now have. Now the 7x7x7 Conway Cube, with 7 holes, should be easy!

5. There is also a large plastic (half) box, with clear plastic pieces. Make sure the box is 7x7x7 and that you have 14 of the 4x4x1 squares. The other pieces should be enough to exactly fill the 7x7x7 cube. This version was invented by Bill Gosper.