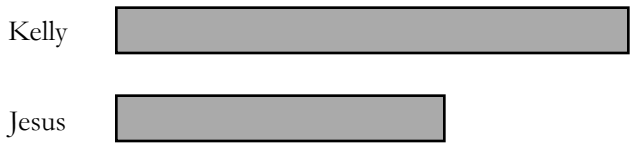
Santa Ana Math Club

November 12, 2011

Name: School: Grade:

Introductory Algebra Notes - The Bob Method:

Jesus and Kelly together have 22 marbles. Kelly has 6 more marbles than Jesus. How many marbles does each person have?

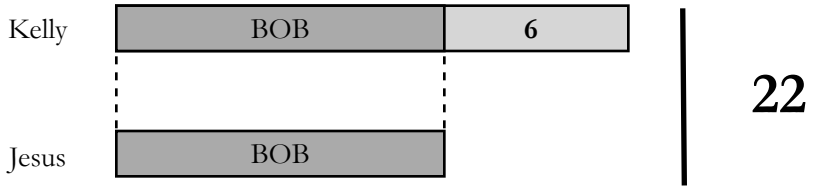


1. The length of the bar represents how many marbles they have. We don't know how long they are, but we do know that Kelly has more marbles, so her bar is longer.

2. Kelly and Jesus have 20 marbles. Kelly and Jesus combined have the same amount of marbles as the other side.

3. Let's name Jesus's bar BOB.

Interesting Fact: Every Bob is exactly the same.

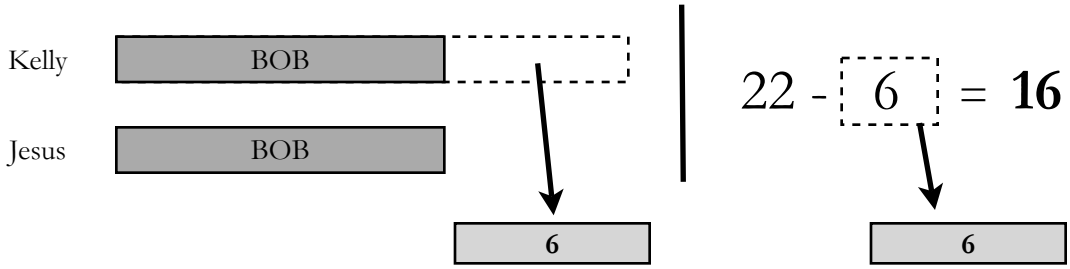


4. Kelly has 6 more marbles than Jesus, so her bar is longer by 6.

5. Her bar is as long as BOB plus 6 more units.

6. Bob only likes other Bobs, so they want to be alone without any numbers. They HATE numbers!

7. We make the 6 leave. But when ones 6 leaves from one side, a 6 leaves from the other side to keep it company.



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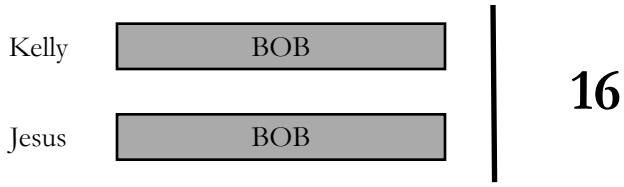
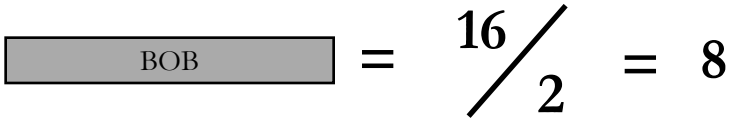


Interesting Fact: Bobs are very fair. They always share.

8. Since there are two Bobs, they split the 16 marbles into two parts. Each Bob gets 8 marbles.



9. Jesus had the same amount of marbles as one Bob. Jesus then has 8 marbles.

Kelly has 6 more marbles than one Bob. She has 14 marbles.

Sample Bob Problems:

1. Sandra and Lesley both like lollipops. Sandra has 6 more lollipops than Lesley. Together, they have 30 lollipops. How many lollipops does Lesley have?

2. Paola and Alexa went to the beach. They both found sea glass and put it in one bucket. In the bucket, they had 48 pieces of sea glass. Paola found 12 more pieces than Alexa. How many pieces did Paola have?

3. Enrique and Alfredo brought candy to lunch. Enrique had two more than twice as many pieces of candy than Alfredo. Together, they put all of their candy in a bag. In that bag, there were 23 pieces. How much candy did Enrique have to begin with?

Challenge: Nancy, Zeus, and Jasmine all like to read. In a certain month, the three of them combined read 20 books. Each person read different books. Jasmine and Zeus read the same number of books. However, Nancy read 4 more books than Zeus and Jasmine of them combined. How many books did each of them read?

Challenge: Jorge and Hector like to play with marbles. Jorge had 7 more than four times the number of marbles that Hector had. Together they had 37 marbles. How many marbles did Jorge have?

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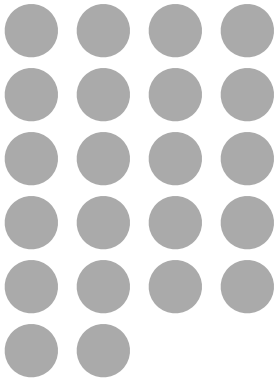
Introductory Algebra Notes - Visual Method:

Jesus and Kelly together have 22 marbles. Kelly has 6 more marbles than Jesus. How many marbles does each person have?

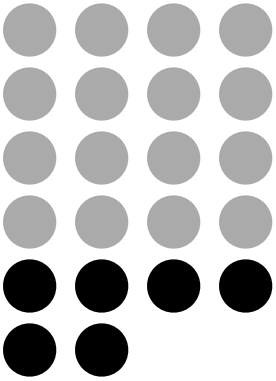


1. The twenty-two circles represent the 22 marbles that Jesus and Kelly have together.

2. Kelly has 6 more than Jesus, so ...

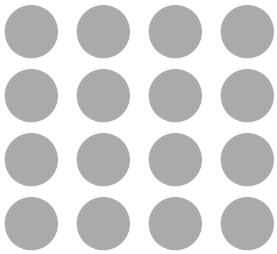
Jesus's marbles + 6 marbles = Kelly's marbles

However many marbles Kelly has is 6 more than Jesus.



3. Kelly has 6 more marbles than Jesus. Take away 6 marbles from Kelly's share, and she now has the same number of marbles as Jesus.

All the marbles look the same, so we can't tell which marbles are hers. Instead, we just take 6 away from their combined bag of marbles.

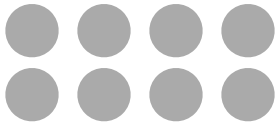


4. There are only 16 marbles left. These marbles are Jesus's and Kelly's.

Important: We took away 6 of Kelly's marbles already. Since Kelly only had 6 more marbles than Jesus to begin with, she now has the same number of marbles **in the bag** as Jesus.

The marbles in the bag represent Jesus's marbles plus Kelly's marbles minus 6.

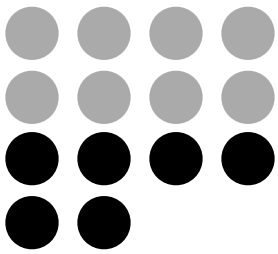
Jesus



5. We split the remaining marbles in the bag into two parts.

6. One pile is Jesus's, the other is Kelly's. However, we took 6 marbles away from the bag in step 3, so we need to return those. **Who do they go to? Kelly**, because she has 6 more than Jesus. So we add the 6 marbles that we took back to her pile.

Kelly



Kelly has 14 marbles, while Jesus has 8.

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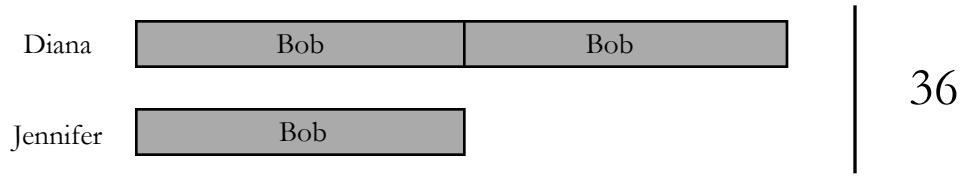
Name: School: Grade:

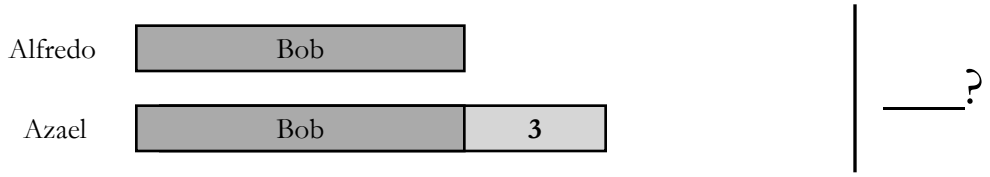
Introductory Algebra Notes - Practice Bob Problems:

Solve the problems starting from the Bob setup given to you.

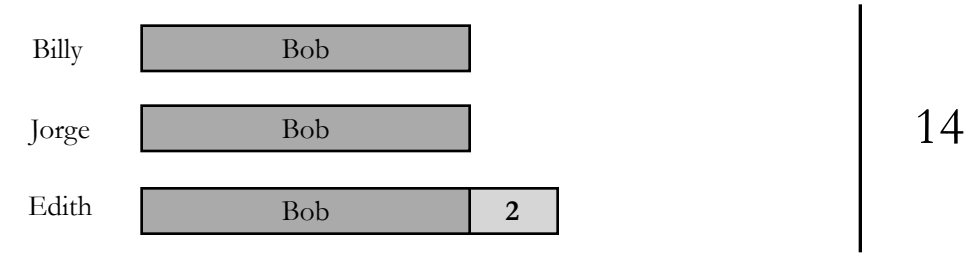
Diana and Jennifer collect stamps. They put their stamps into the same book. Diana collected twice as many stamps as Jennifer. How many did each collect if there are 36 stamps in the book?



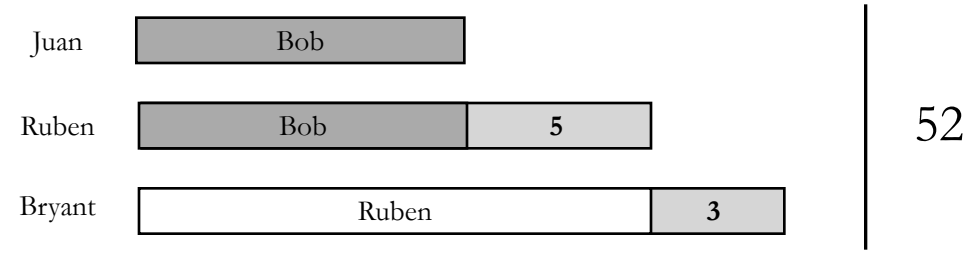
Alfredo and Azael were sharing a baker's dozen of cookies. (**Note:** A baker's dozen is 13. This is a common math competition trick. Know this now instead of missing it later :D) Azael ate three more cookies than Alfredo. How many cookies did Alfredo eat?



Billy. Jorge, and Edith had a movie night. Each person brought several movies on DVD. They watched all of the 14 movies they brought. Billy and Jorge brought the same number of movies, while Edith brought two more movies than Jorge. How many movies did Edith bring?



Challenge: Ruben, Juan, and Bryant were playing with a normal deck of cards. They used all 52 cards. Ruben had five more cards than Juan. Bryant had three more cards than Ruben. How many cards does each person have?



Hint: Make Bryant's bar divided into Bob + something.

Apply the Bob method to Bryant as compared to Ruben.

Santa Ana Math Club

November 12, 2011

Name: Grade:

Introduction Algebra and Functions:

Introduction to Algebra

Terms/Definitions:

Variable: a letter that can be used to represent one or more numbers

What is "x"? (Take notes here:)

Expression vs. Equation

Expression: includes only numbers and/or variables

- Numerical expression: An expression that includes only numbers
- Algebraic expression: An expression that combines numbers and variables by operations (+,--,x,÷)
  - An algebraic expression includes both variables and numerical expressions.

Algebraic equation: a statement by placing an equal sign "=" between two expressions

Term: A single number or a product of numbers and variables in an expression

There are two terms in the expression $x + 3$, and $5x^2 + 7$

There are three terms in the expression $3x^2 - 5x - 2$

There are four terms in the expression $xy + x + 2y - 3$

Constant: a term without a variable. It is only a number (i.e. 5 is a constant)

Coefficient: a number that is multiplied by a variable (in the term 5y, 5 is the coefficient), (the coefficient of the term x is 1)

Monomial: an expression with only one term (5x or 12 or 42x)

Polynomial: an expression with two or more terms

Degree of an expression --

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Adding/Subtracting Expressions:

*To simplify an expression, we combine like terms.

*To solve an equation, we:

1) Combine like terms of variables
2) Separate variables from constants
3) +, , x, or ÷ so that the coefficient of the variable becomes 1.

Example: $2x + 3x = 5x$.

Try it!

Simplify each expression.

1. $3x + 6x$
2. $-3x \quad 6x$
3. $\frac{1}{2}x + \frac{1}{2}x + \frac{1}{2}y + \frac{1}{2}y$
4. $2x^2 + 5 + 4x^2 - 6$

Solve!

Solve each equation for x.

5. $x - 5 = 9$
6. $5x = 40$
7. $x \div 5 = 40$
8. $-12 = -2x$
9. $2x + 5x = 28$
10. $7x = 9x + 24$
11. $3x - 2x + x + x - 5x = 324$
12. $2x + 15 = -3x$
13. $5x + 12 = 3x$
14. $-6x + 8 = 2x$
15. $3x + 2x + 11 = 36$

Santa Ana Math Club

November 12, 2011 - Homework

Name: Grade:

Solve these problems. For many of them, the Bob method might be helpful.
Consult your notes to help you and ask your teachers if you get stuck on any problem.
Don't worry if you cannot complete all of the problems. :)

Simplify these expressions:

1. $3x + 4 + 7x - 5 =$
2. $30 - (4x + 30) =$
3. $2 + 8x + (4x + 3) =$
4. $4x^2 + 3x^2 =$
5. $x^2 + 2x + 3x + 3x^2 =$

Solve these problems (what does x equal?):

1. $3x + 4 + 7x - 5 = 19$
2. $7x + 5 = 12$
3. $9 + 4x + 3 = 20$
4. $2x + 4x + 5 = 41$
5. $x^2 + 3x - 3x = 16$

1. If there are x black ducks and 14 yellow ducks, and there are 24 ducks in total, how many black ducks are there?

2. If there are 58 photos in Bo & Bob's slideshow and 28 of them are Bob's, how many pictures are Bo's?

3. In the park, there are d amount of ducks. Every minute, one duck flies away. How many ducks are left after 10 minutes? Write your answer in terms of d.

4. If Problem Set A takes 25 minutes to solve and Set B takes 30 minutes, how many hours does it take Mr. Mole to finish 2 Problem Set A's and 1 Problem Set B?

5. A dozen cupcakes take 3 times longer to make than a dozen donuts. It takes Mark 30 min. to make a dozen donuts. How long will it take him to make 2 dozens cupcakes?

Santa Ana Math Club

November 12, 2011 - Homework

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6. Bob is 2 years younger than Nelson. Nelson and Bob's age combined is 32. How old is Bob?

7. Aaron has a certain amount of cash. If Andrew has 5 more dollars than Aaron does, and together they have $10.50, how much money does Andy have?

8. Mrs. Piggy bakes brownies and gives a third of them to her pig. Then she gives 8 to her neighbor the elephant. If Mrs. Piggy has none left, how many brownies did she make?

9. If Dog gets $10 per hour for digging and Cat gets $8.75 for the same job, how much more money does Dog earn than Cat after 12 hours?

10. At a school field trip, for every 10 students there is one adult supervisor. If "x" is the number of students, write the number of adult supervisor in terms of "x".

<u>Challenge Problems</u>

11. 25 less than a number is 40. Find the number.

12. Five times a number is 95. Find the number.

13. Eight times the difference between a number and 15 is 40. Find the number.

14. 12 less than 4 times a number is twice the number. Find the number.

Visualizing Algebra

Orange County Math Circle
http://www.ocmathcircle.org/
Alex Zivkovic
Anne Chen

1. Introduction

The activity described below was first implemented in the Orange County Math Circle's Santa Ana Math Club program. The problem set was delivered to students in grades 4 through 8, and contains material that is meant to challenge students at varying math levels.

Please review these instructional pages to see how a formal lesson plan can be developed from our problem set. This packet can be used to accomplish a variety of things, so lesson plans targeted at students with varying mathematical levels can be created.

For this problem set, students at higher or lower mathematical levels can enjoy solving the problems since they fall outside of the school curriculum.

2. Start a discussion about algebra

Depending on the age level of the students, this step can be skipped.

Discuss how you can solve for unknowns. What are unknowns? Use "dozen" and other words that describe quantities to explain this concept.

3. Walk through the first algebra problem (Bob Method - page 1/2)

Work through these problems following the "Bob Method" adapted from the Singapore Math Book. Walk the students through this problem.

Skip this problems and go back to them after step 5. Have them draw out these problems using the Bob method (problems 1-3). If your students are unusually advanced, have them try out the Bob method with 3 people (as in the first challenge) or with a "multiple bob" problem. If not, go back to these problems after seeing the other method.

4. Walk through the first algebra problem (Visual Method - page 3)

This problem is the same as the first one, but instead of using an abstract term of "Bob" to describe x marbles, now you will show the marbles grouped into Bob-units.

If students are confused by the Bob Method or this method, show both at the same time. Again, draw the comparison to "pair" or "dozen" or "triple" (words that describe quantities), explaining that "Bob" or "x" describes a number as well, except unlike concrete English words, we do not know what number they stand for.

5. Applying what they learned to individual work (Page 4, go back to page 2)

These problems already set up the Bob method. Use these pages to allow students to work alone. If students are confused, have them underline the numbers in the word problems to show how the bars show the same problem as the word problem.

If students finish these, have them go back to page 2 to draw their own bars out for problems 1-3. The challenge problems are difficult because they introduce using multiple Bobs in one bar and having more than two people in a problem, so to help them, have them reference the bars in page 4 as a model.

6. Introduction to Algebraic Notation (page 5)

Some students may be familiar with algebraic notation, but if not, this is a good moment to try to introduce this concept. The concepts introduced will be familiar to 6th, 7th, and 8th graders, but younger students might struggle.

Note: This is simply an introduction. We have been teaching this to students starting in 4th grade, and when we reteach it to them as 5th graders, they understand it much better. The key is to simply present the information, so it is okay if they do not understand all of it.

Many terms are here which will prove useful review for older students. For younger students, try to explain terms only. Feel free to skip monomial, polynomial, constant, coefficient. Just explain first degree expressions with one variable.

7. Manipulating Algebraic Notation (page 6)

For younger students, merely try to have them add variables. They do not need to solve equations that require addition and division.

8. Homework (pages 7 - 8)

The homework covers algebraic notation and helps bridge the gap between Bob problems and actual algebraic problems. Have them use whatever method they like. Only give this homework to the older students, because it may be discouraging to younger students.

For the younger ones, instead bring a review sheet to the next class to review the concepts (single term expressions like 3x or 5y, first degree addition equations like $x + 3 = 9$, and Bob problems with bars already drawn).