What is a matrix? What good are matrices?

Consider the 4 problems below. What is changing from problem to problem?

1. A furniture store makes $30 profit per table on each of the 17 tables it sells. How much was the profit from selling tables?

2. A furniture store makes $30 profit per table it sells and $7 profit per chair it sells. In January, the furniture store sold 17 tables and 32 chairs. How much profit did it make from selling tables and chairs?

3. A furniture store makes $30 profit per table it sells and $7 profit per chair it sells. The store spends 1.5 hours of time preparing and selling each table and 0.5 hours preparing and selling each chair. In January, the furniture store sold 17 tables and 32 chairs. What was the profit from selling tables and chairs? How much time was spent preparing and selling tables and chairs?

4. A furniture store makes $30 profit per table it sells and $7 profit per chair it sells. The store spends 1.5 hours of time preparing and selling each table and 0.5 hours preparing and selling each chair. In January, the furniture store sold 17 tables and 32 chairs. In February it sold 21 tables and 42 chairs. What was the profit from selling tables and chairs in each of the two months respectively? How much time was spent preparing and selling the furniture in each of the two months respectively?

None of the problems above are too difficult but the information presented is doubled from one problem to the next.

1. This is a simple multiplication: 30$/Tables*17 Tables = 510 Tables
2. This problem combines two products by adding:
   $30/T*17T + $7/C*32C =
3. The third problem seeks the answer to two problems. The first is from #2 and the second is similar, but the result is measured in hours instead of $.
4. Again, this problem doubles the complexity. We need to duplicate what was done in #3 for February.

Matrices provide a systematic way of organizing information. Let's see how we can apply them to the problems above.

1. This is a simple multiplication and could be organized in the smallest of possible matrices, but let's not bother with that.
2. The information about the rates of profits is organized into a row matrix: [30 7]. The information about the quantities of tables and chairs is

organized into a column matrix: $\begin{bmatrix}17\\32\end{bmatrix}$. The combination we need is expressed as a "product" of the matrices: $\begin{bmatrix}30 & 7\end{bmatrix}\begin{bmatrix}17\\32\end{bmatrix} = 30 * 17 + 7 * 32 = 734$.

3. This problem really has two parts, each similar to #2. The rates in the first part are about profit while the rates in the second part are about time for preparation. We can express the second part just as the first: $\begin{bmatrix}1.5 & 0.5\end{bmatrix}\begin{bmatrix}17\\32\end{bmatrix} = 1.5 * 17 + 0.5 * 32 = 41.5$. We have the column matrix, so it isn't too much of a stretch to put the two "rate matrices," each in the form of a row, into a single matrix and multiply by the quantity column: $\begin{bmatrix}30 & 7\\1.5 & 0.5\end{bmatrix}\begin{bmatrix}17\\32\end{bmatrix} = \begin{bmatrix}30 * 17 + 7 * 32\\1.5 * 17 + 0.5 * 32 = 41.5\end{bmatrix} = \begin{bmatrix}734\\41.5\end{bmatrix}$. So there wasn't any less calculation, but the matrices are helping us organize the information.
4. In the final problem we are jumping from 2 calculations to 4. Since we are adding more quantity information, we will expand the matrix equation. In this case, we add a column to the quantity matrix. $\begin{bmatrix}30 & 7\\1.5 & 0.5\end{bmatrix}\begin{bmatrix}17 & 21\\32 & 42\end{bmatrix} = \begin{bmatrix}734 & 30 * 21 + 7 * 42\\41.5 & 1.5 * 21 + 0.5 * 42\end{bmatrix} = \begin{bmatrix}734 & 924\\41.5 & 52.5\end{bmatrix}$.

So, not only have we used matrices to organize numbers into rows and columns, but we have multiplied them in meaningful ways to process those numbers. Of course, this doesn't apply to just furniture. You will see many more applications as well as the properties of arithmetic of matrices.