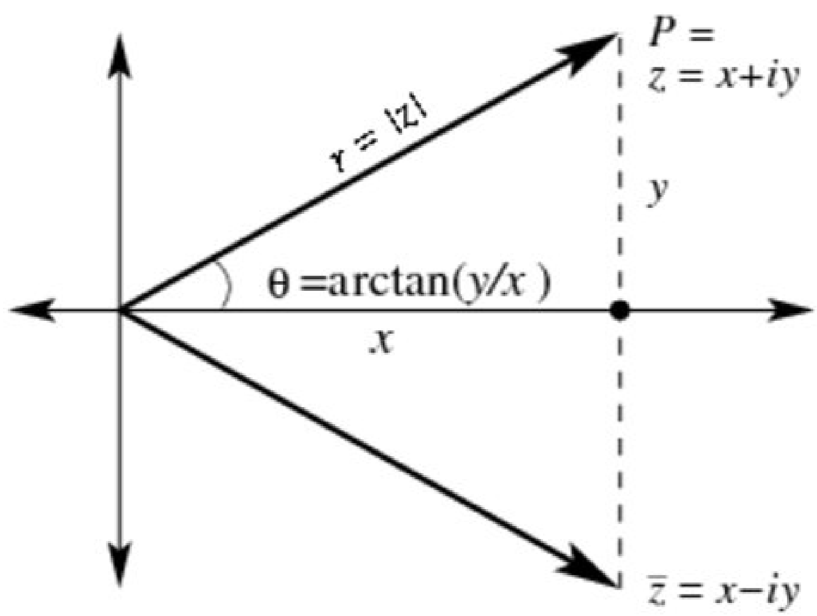
Exploring Kleinian Groups

Circle on the Road

Complex Numbers



A complex number z can be described either via its real and imaginary parts x and y or via its modulus r and argument θ. The complex conjugate $\overline{z} = x - iy$ is obtained by reflecting in the real axis.

Always remember that $i^2 = -1$. For example,

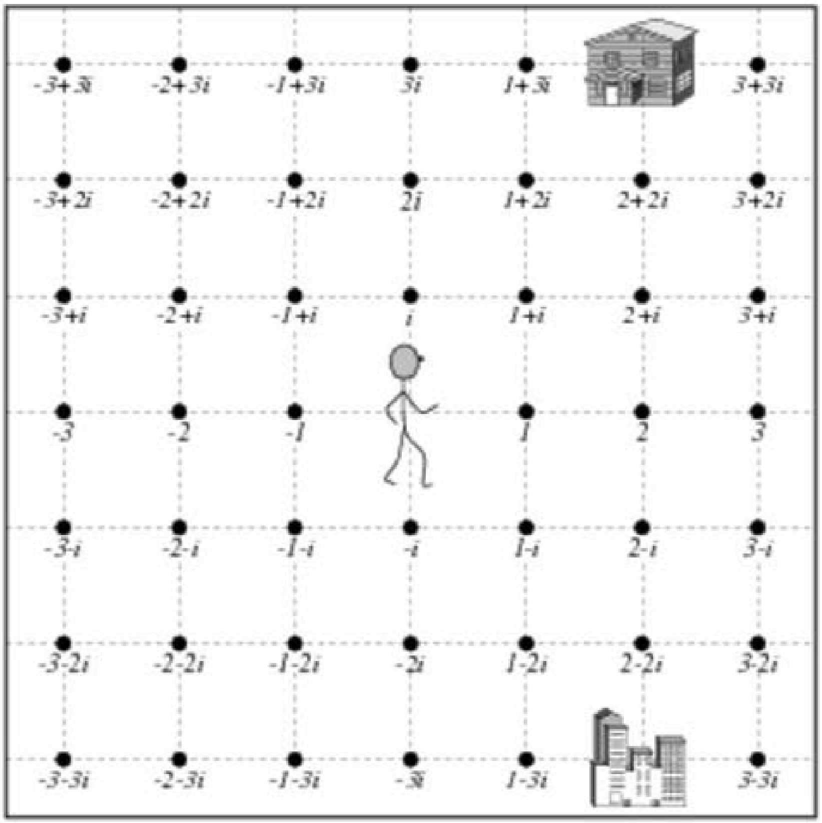
$$(3+4i)+(2-3i)=(3+2)+(4i-3i)=5+i$$

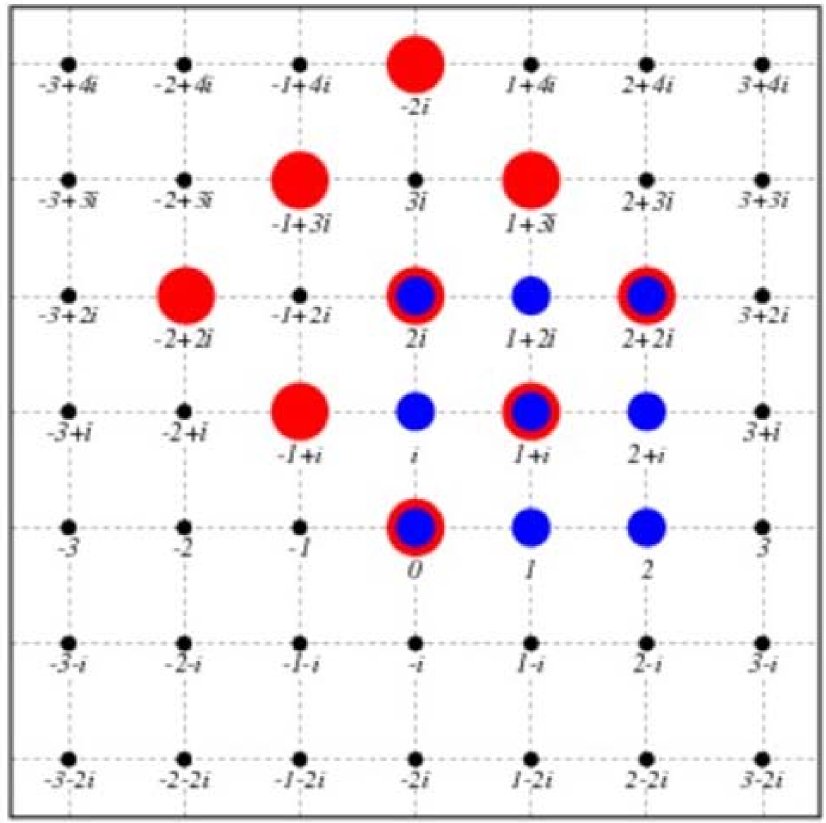
and

$$(3+4i)(2-3i)=6-9i+8i-12i^2=6-i+12=18-i$$

and

$$\frac{1}{2+i}=\frac{2-i}{2^2+1^2}=\frac{2}{5}-\frac{i}{5}.$$


-3+3i -2+3i -1+3i 3i 1+3i 3+3i
-3+2i -2+2i -1+2i 2i 1+2i 2+2i 3+2i
-3+i -2+i -1+i i 1+i 2+i 3+i
-3 -2 -1 1 2 3
-3-i -2-i -1-i -i 1-i 2-i 3-i
-3-2i -2-2i -1-2i -2i 1-2i 2-2i 3-2i
-3-3i -2-3i -1-3i -3i 1-3i 3-3i


-3+4i -2+4i -1+4i -2i 1+4i 2+4i 3+4i
-3+3i -2+3i -1+3i 3i 1+3i 2+3i 3+3i
-3+2i -2+2i -1+2i 2i 1+2i 2+2i 3+2i
-3+i -2+i -1+i i 1+i 2+i 3+i
-3 -2 -1 0 1 2 3
-3-i -2-i -1-i -i 1-i 2-i 3-i
-3-2i -2-2i -1-2i -2i 1-2i 2-2i 3-2i

All Euclidean symmetries may be expressed using complex numbers as

$$T(z) = az + b \qquad \text{or} \qquad T(z) = a\bar{z} + b$$

where $z = x + iy$ where x, y are real and $i = \sqrt{-1}$, and a and b are both complex constants with $a \neq 0$.

If $|a| = 1$, T is a rigid motion.

Example: $T(z) = \boxed{}\,z$ is counterclockwise rotation by $\boxed{}$ together with a dilation by a factor of $\boxed{}$.

Every Kleinian symmetry (usually called a Möbius transformation) may be expressed as

$$T(z) = \frac{az + b}{cz + d} \qquad \text{or} \qquad T(z) = \frac{a\overline{z} + b}{c\overline{z} + d}$$

where a, b, c, d are complex numbers such that $ad - bc \neq 0$.

Example: $T(z) = \dfrac{1}{\overline{z}}$ is called **inversion** in the unit circle.