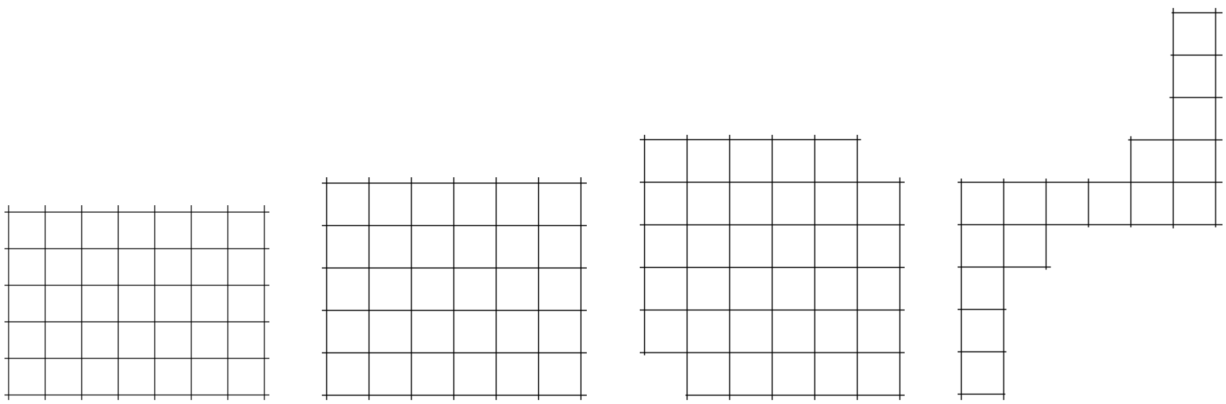
Terrific Tiles ...

1. For each of the above figures, find a domino tiling, or show that one does not exist.



2. Is it possible to remove just one square from a 5×5 checkerboard and then tile the resulting region using dominos? If so what are the possibilities for the square that is removed?

3. Is it possible to remove just one square from a 8×8 checkerboard and then tile the resulting region using 1×3 rectangles? If so what are the possibilities for the square that is removed?

4. Put a number on one corner of a domino on a checkerboard. Then work your way counter-clockwise around the domino writing additional numbers according to the following rule: if there is a dark square on the left side of the edge that is being followed, add $\frac{1}{4}$ to the previous number. If there is a light square, subtract $\frac{1}{4}$. What happens when you go all the way around a domino? What does this have to do with the Darn Good commutative law, or should I say the Domino Group?

5. If the above process is done to every domino in a tiling the resulting values will be called the *tiling function* of the region. Compute the tiling function associated to each way of tiling a 2×2 grid.

6. Is it possible to compute the values of the tiling function of the boundary of a region without knowing the tiling inside of the region? If so, do so for the regions in the above figure.

7. Compute the tiling functions for the regions that you tiled above.

8. Given any tiling function, show that it is possible to change the tiling so that the maximum of the associated tiling function is on the boundary. do this process with at least one of your examples. Where would a tile have to be placed given that the maximum value is on the boundary? Show that this gives an algorithm that produces a tiling of a given region, or proves that the region can not be tiled.

9. Draw two rather distinct tilings of the same grid region, then superimpose them. (Use line segments to denote the dominos.) What do you notice? Use this to show that you can convert one tiling into the other by a sequence of 2×2 grid moves.

10. In how many different ways can a $2 \times n$ grid be tiled with dominos.

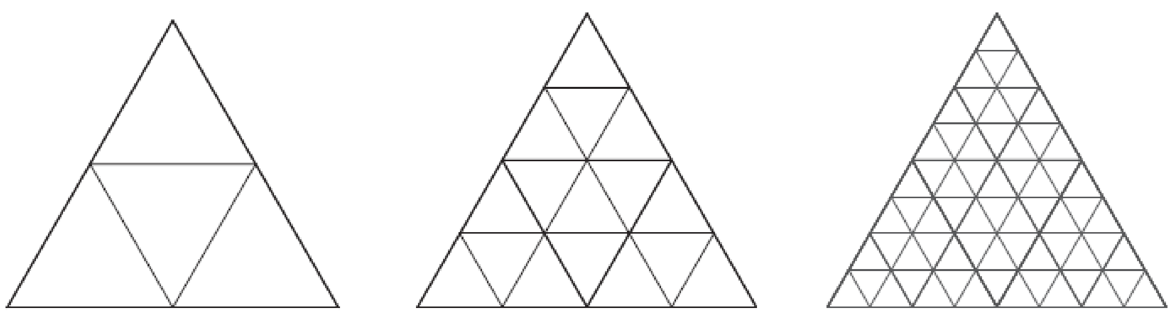
A *bent triomino* is made by removing one square from a 2×2 grid.

1. Is it possible to remove a square from a 4×4 grid and tile the remainder with bent triominos? If so what are the possibilities for the square that is removed? What about a 8×8 grid? a 16×16 grid?

2. Which square boards of size $n \times n$ from which one square has been removed can be tiled with bent triominos? (Hint: A 5×5 board is special.)

3. Which $n \times m$ rectangular boards can be tiled with bent triominos?

Write $T(1)$ for a triangular board of side-length 2 which is subdivided into equilateral triangles each of side length 1, $T(2)$ for the board of side-length 4, $T(3)$ for the board of side-length 6, etc. If a triangle shares one (or two) of its sides with the large triangle, then it is called an *edge triangle*. If it shares two of its sides with the large triangle, then it is called a *corner triangle*. A *triangular triomino* is a tile consisting of three adjacent triangles.



1. For which n is it possible to tile the $T(n)$ board with triangular triominos after any (one) corner triangle is removed? an edge triangle is removed? an interior triangle is removed?

2. For which n is it possible to tile the remaining board with triangular triominos after all the corner triangles and any other (just one more) triangle are removed from $T(n)$?