Owls and Voles

Overview

There is a delicate balance between predators and prey in an ecosystem. In this activity, students work in groups to play the role of a family of barn owls. They determine how many voles their family can eat without destroying their food supply. Based on actual data about barn owls, students use their model to estimate the number of voles needed to support a barn owl family. Students use algebraic equations and a spreadsheet to make quantitative predictions. Students later modify the model based on facts they learn about owls and voles.

Prerequisites

- Familiarity with the slope-intercept form of a line.
- Ability to make an x, y table given the equation of a line.

Goals

- Students will strengthen and contextualize their understanding of algebraic expressions.
- Students will experience the interplay between a scientific model and the reality it represents as they explore the behavior of a simple model, identify ecological inaccuracies in the model, modify the model to correct for one of these inaccuracies, use their models to make quantitative predictions.

Materials and Preparation

- A way to wash hands (if using food items)
- 25 Gummy bears, raisins, sunflower seeds, or other items per person
- 1 plate for each group of four
- 1 sandwich bag for each person
- Pencil and paper (or notebook) for each person
- 1 copy of the handouts for each person
- Access to a spreadsheet program for each group
- Access to the internet or a source of facts about owls and voles
- A blackboard, white board, or overhead projector
- Owl pellets and bone charts if desired

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References

1. American Forest Foundation. *Project Learning Tree: Environmental Education Activity Guide.* 1993; Pages 43-47.
2. Edelstein-Keshet, Leah. *Mathematical Models in Biology (Classics in Applied Mathematics).* SIAM; 2005.
3. Hoppensteadt, F. *Mathematical theories of populations: demographics, genetics and epidemics.* SIAM, 1975.
4. Kaplan, Daniel and Leon Glass. *Understanding Nonlinear Dynamics (Textbooks in Mathematical Sciences).* Springer-Verlag; 1995.
5. Murray, James D. *Mathematical Biology (Interdisciplinary Applied Mathematics, Volume 17).* Springer-Verlag; 2002.
6. Rubinow, S. I. *Introduction to mathematical biology.* John Wiley, 1975.

Related Common Core Math Standards

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

8.F.4 Construct a function to model a linear relationship between two quantities.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g. where the function is increasing or decreasing, linear or nonlinear).

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Patterns include clustering, outliers, positive or negative association, linear or nonlinear association.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit.

HS.A-SSE.1 Interpret expressions that represent a quantity in terms of its context.

HS.A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Apply linear, quadratic, rational, and exponential functions.

HS.A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

HS.F-BF.1 Write a function that describes a relationship between two quantities.

HS.F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

HS.F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

HS.F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

HS.S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

Tables of Values and Cell References

Answer the numbered questions in bold on another page (you may type your answers if you wish).

Your first task is to use the spreadsheet to generate a table of values for the equation $y = 3x + 4$. Your table should include all integer x-values from -10 to 10.

- In cell A1, type "x" as a label for the x column.
- In cell B1, type "y" as a label for the y column.
- In cell A2, type the number "-10".
- In cell A3, type "=A2+1" and press "Enter".

(1) What does the last step do?

- Highlight cell A3 either by navigating with the arrows or by clicking on the cell.
- Copy cell A3 by holding the "Ctrl" key and pressing "C". (Alternatively, you can use the mouse to select "Edit" and then "Copy" in the menu at the top of the screen.)
- Highlight cells A4 through A22. You can use the mouse to highlight these cells or you can start at cell A4 and use the "Shift" and down arrow key until the highlighted region includes A22.
- Paste by holding the "Ctrl" key and pressing "V". (Alternatively, you can use the mouse to select "Edit" and then "Paste" in the menu at the top of the screen.)
- Press the "Esc" key to make cell A3 stop flashing.

(2) What did you just do? Highlight one of the filled cells in column A and look at the formula box at the top of the screen. Look at some other cells in that column. How is the formula working? What happened when you copied cell A2 and pasted the result?

- In cell B2, type "=3*A2 + 4".
- Copy cell B2.
- Highlight cells B3 through B22 and paste.
- One at a time, highlight several cells in column B and look at the formula box at the top of the screen.

(3) What is happening in each row?

(4) What would you need to change to produce a table for the formula $y = -2x+7$?

- Generate the table for this new formula. Remember to copy the new formula into all of the cells in column B.

(5) In the formula $y = 5x + 1$, what is the slope and what is the y-intercept?

- In cell D1, type the letter "m" to label the slope.
- In cell D2, enter the slope for this formula.
- In cell E1, type the letter "b" to label the y-intercept.
- In cell E2, enter the y-intercept for this formula.

Earlier, we copied the formula $y = -2x + 7$ into all of the cells in column B. To change the slope and the y-intercept to the new values, we would need to copy that formula into all of those cells. However, if we know ahead of time that we are going to be changing the slope and y-intercept, we can set things up in the spreadsheet so it is easy to make these changes.

- Highlight cell B2.

- Type "= D2 * A2 + E2" and press "Enter". Notice the location of the dollar signs.
- Highlight cell B2 and copy it.
- Highlight cells B3 through B22 and paste.

Try changing the values of the slope and y-intercept in cells D2 and E2. Notice how the table of values quickly adjusts after each change.

(6) When creating a spreadsheet formula, when should you use dollar signs in front of the cell labels and when should you write the labels without dollar signs?

- Insert a line graph that shows the values you produced. The easiest way to do this is to highlight all of the number values in columns A and B, then click "Insert" and select the "Scatter" graph icon.
- To adjust the grid lines, select the "Layout" tab under "Chart Tools" and then select "Gridlines".
- You may wish to remove or edit the chart title and series label. You can move, rescale, or resize the chart by clicking outside the chart and then clicking and dragging the corners or edges of the boundary box.

(7) Preview and print your final spreadsheet.

Fish in Peaceful Lake

Write or type your solutions. Include a print-out of your spreadsheet.

Every week, the people around Peaceful Lake catch 100 fish from the lake to eat. (Suppose that no new fish are born and that no fish are dying for other reasons.) Let F_w stand for the number of fish in the lake one week, and let F_{w+1} stand for the number of fish in the lake the following week.

(1) Write an equation that expresses F_{w+1} in terms of F_w.

Suppose that there are 10,000 fish in Peaceful Lake to begin with. Create a spreadsheet to show how many fish there will be in the lake each week for 15 weeks. Title the spreadsheet "Fish in Peaceful Lake". Label one column with the word "week" and the other column with the words "number of fish". Number the weeks 0 through 15. In the cell under the words "number of fish", you should enter the number 10,000. Enter a formula to get the spreadsheet to show the number of fish that goes in the next cell down. Copy this formula and paste it into the other cells in the column.

(2) How many fish will be in the lake at the end of 15 weeks according to this model?

(3) Make a graph showing the number of fish in Peaceful Lake over time. What kind of function does the graph show?

(4) Write an equation for the the number of fish in Peaceful Lake as a function of the week number.

(5) Use your equation to find the number of fish after 23 weeks. Show your work. Check your answer by extending the table to week 23.

Click on the edge of the graph so that it is highlighted. Under "Chart Tools" and "Layout", select "Trendline". Select "More Trendline Options". Select the kind of function that best matches your data, and at the bottom of the dialog, select the options to "Display Equation on chart" and "Display R-squared value on chart". Click "Close" and then move the box with the equation and R-Squared value to a place where it is easy to read.

(6) How does the equation compare with the one you found?

(7) What is the R-squared value? What does that mean?

(8) Print your spreadsheet.

Bacteria in a Petri Dish

Every hour, each bacterium in a petri dish splits into two bacteria so that the total number doubles. Let B_h stand for the number of bacteria in the dish one hour, and let B_{h+1} stand for the number of bacteria in the dish the following hour.

(1) Write an equation that expresses B_{h+1} in terms of B_h.

Suppose that the petri dish has 1 bacterium at hour 0. Use a spreadsheet to determine how many bacteria will be in the dish after 24 hours. Include an appropriate title and column headings. Set up the spreadsheet with appropriate column labels, number of rows, and formulas.

(2) How many bacteria will be in the dish after 24 hours?

(3) Make a graph showing the number of bacteria over time. What kind of function does the graph show?

(4) Write an equation for the number of bacteria as a function of the hour number.

(5) Use your equation to find the number of bacteria in the dish after 24 hours. Show your work.

Choose an appropriate trendline for your graph and show the Equation and R-squared value on the graph.

(6) How does the equation compare with the one you found?

(7) What is the R-squared value? What does that mean?

(8) Print your spreadsheet.

Rabbits and the Floor Function

Let R_m stand for the number of rabbits one month and let R_{m+1} stand for the number of rabbits the next month. Suppose that we start with 6 rabbits in month 0.

(1) What is a possible biological meaning of the following mathematical model?
$R_{m+1} = R_m + \frac{1}{2}R_m$.

One difficulty with this model is that when R_m is odd, the number of new rabbits will not be a whole number. We can fix that problem with the floor function. The *floor function* is a way to round numbers down to the next integer. For example $\lfloor 4.7 \rfloor = 4$, $\lfloor 0.2 \rfloor = 0$, and $\lfloor 5 \rfloor = 5$.

Set up a spreadsheet to figure out how many rabbits there will be after one year.

- Label column A with the word "month" and label column B with "number of rabbits".
- In column A, number the months from 0 up to 12.
- In cell B2, type "6" to represent the starting number.
- In cell B3, type "=FLOOR((B2 + 1/2 * B2),1)"
- Highlight and copy cell B3.
- Paste into the rest of column B (through month 12).

The 1 at the end of the FLOOR function is the cue to the computer to reduce the value down to the next lowest whole number. If we enter a 2 at the end of the FLOOR function instead, it will reduce the value down to the next lowest even number. If we enter a 4 at the end, the FLOOR function will reduce the value down to the next lowest multiple of 4. If we enter 0.5, the FLOOR function will reduce down to the next lowest half integer.

(2) How many rabbits will there be after one year according to this model?

(3) Make a graph showing the number of rabbits over time. What kind of function does the graph show?

Choose an appropriate trendline for your graph and show the Equation and R-squared value on the graph.

(4) What was the equation for the trendline?

(5) Use this equation to find out how many rabbits there would be after 2 years. Show your work.

(6) Compare this value with what you get by extending your table. Is the estimate you found from the equation close in any sense? Why do you think there is a difference between the answers?

(7) Print your spreadsheet.

Owls and Voles Simulation Instructions

In this activity, your group will be a family of barn owls. You will be working with a mathematical model of how owls and voles interact. The model is simpler than the way things work in real life, but it has some realistic features. Later, you will have the opportunity to change the model to be a bit more realistic.

Your group needs the following items:

- Vole Population tracking page and a pencil
- A plate that symbolizes the meadow where your family of owls hunts.
- A bag of "meadow voles".
- 1 sandwich bag for each team member.

Read the following directions before you begin the activity.

- Place 5 times as many voles in the meadow as there are owls in your family. Write this starting number of voles on the Vole Population tracking page next to "Month 1".
- Each month your family of owls eats some voles. (You should not actually eat the voles during the activity. Put them in your sandwich bag to eat later.) Each owl can eat as many voles as he or she likes during eating time. Any owl who does not eat at least one vole will die of starvation. Your family of owls needs to eat as much as possible over time so that you can produce baby owls.
- After the eating time, you should figure out how many voles were eaten in all. This number goes next to "Voles Eaten". Subtract to find the number of voles remaining. (This should match the number of voles left in the meadow.)
- Next, the voles left in the meadow have babies. Each pair of voles will produce one baby, so divide the number of remaining voles by 2 and write the result next to "Voles Born". If the number of voles is odd, subtract one before dividing by 2. So if there are 6 voles left in the meadow, then 3 baby voles are born. If there are 9 voles left in the meadow, 4 baby voles are born. Put the new baby voles into the meadow and then add the number of baby voles to find the total voles for Month 2.
- This process will repeat for 6 months and then stop. Your family can only support babies if you have eaten enough and if there are enough voles left to provide for additional owls. When you are finished with the 6-month simulation, use the table below to see whether your family can support a baby owl. BOTH requirements in each box must be met to support babies.

Number of owls in your family	Requirements for 1 Baby	Requirements for 2 Babies	Requirements for 3 babies
3	At least 14 voles left At least 21 voles eaten	At last 17 voles left At least 24 voles eaten	At least 20 voles left At least 27 voles eaten
4	At least 17 voles left At least 28 voles eaten	At last 20 voles left At least 32 voles eaten	At least 23 voles left At least 36 voles eaten
5	At least 20 voles left At least 35 voles eaten	At last 23 voles left At least 40 voles eaten	At least 26 voles left At least 45 voles eaten

Vole Population

Month 1
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 2
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 3
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 4
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 5
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 6
Voles Eaten − ______________

=

Voles Born + ______________

=

Voles Remaining: ______________

Total Voles eaten: ______________

Baby Owls Born: ______________

Vole Population

Month 1
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 2
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 3
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 4
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 5
Voles Eaten − ______________

=

Voles Born + ______________

=

Month 6
Voles Eaten − ______________

=

Voles Born + ______________

=

Voles Remaining: ______________

Total Voles eaten: ______________

Baby Owls Born: ______________

Vole Population

Month 1

Voles Eaten −
___
=

Voles Born +
___
=

Month 2

Voles Eaten −
___
=

Voles Born +
___
=

Month 3

Voles Eaten −
___
=

Voles Born +
___
=

Month 4

Voles Eaten −
___
=

Voles Born +
___
=

Month 5

Voles Eaten −
___
=

Voles Born +
___
=

Month 6

Voles Eaten −
___
=

Voles Born +
___
=

Voles Remaining: ___

Total Voles eaten: ___

Baby Owls Born: ___

Vole Population

Month 1

Voles Eaten −
___
=

Voles Born +
___
=

Month 2

Voles Eaten −
___
=

Voles Born +
___
=

Month 3

Voles Eaten −
___
=

Voles Born +
___
=

Month 4

Voles Eaten −
___
=

Voles Born +
___
=

Month 5

Voles Eaten −
___
=

Voles Born +
___
=

Month 6

Voles Eaten −
___
=

Voles Born +
___
=

Voles Remaining: ___

Total Voles eaten: ___

Baby Owls Born: ___

Owls and Voles Simulation Questions

After completing the first simulation, answer these questions.

(1) What hunting strategy did your group use?

(2) Did your entire family survive for six months?

(3) How many voles did you collectively eat over the six months?

(4) How many voles were left in the meadow?

(5) Write the hunting strategy that your family will use for the next simulation. How can your family eat as many voles as possible over the six months without destroying your food supply?

(6) Try your strategy. What were the results this time?

(7) Record your answers to the following questions in the chart on the next page.

Suppose that your family only contained 3 owls and that each owl must eat 1 vole per month. What is the smallest number of voles that the meadow could contain to feed the family of 3 owls sustainably?

Suppose that your family contained 4 owls and that each owl must eat 1 vole per month. How many voles would need to be in the meadow in this case? What if your family contained 5 owls?

What is the pattern in this table? If the number of owls is represented by the letter n, what expression tells you the number of voles that would need to be in the meadow?

What if each owl had to eat 2 voles each month to survive instead of just 1? Now how many voles are required to feed a family of 3? How many voles are needed to feed a family of 4? What about a family of 5?

Answer these questions for the case where each owl must eat 3 voles each month.

What if we use the letter v to represent the number of voles eaten each month? Complete the final column using the appropriate expressions.

Minimum Number of Voles Needed

Number of owls in the family	If each owl eats 1 vole	If each owl eats 2 voles	If each owl eats 3 voles	If each owl eats v voles
3				
4				
5				
n				

Modeling the Simulation

(1) Write a formula for how the vole population changes over time in our simulation.

Use a spreadsheet to implement this model. Save some space at the top of your spreadsheet to enter constants such as the number of voles each owl will eat per month, the number of owls in the family, and the starting vole population.

Use several different columns to represent different components of the model such as the month number, each month's starting population, the voles eaten, and the voles born.

(2) Write the formulas you are using in your spreadsheet here.

Check whether your spreadsheet model gives you the same answers you found in your chart.

(3) Make a printout of your spreadsheet.

Inaccuracies in the Model

We would like to estimate the true number of voles needed to support a family of owls. One major problem with our simple model is that barn owls need to eat more than one meadow vole per month to survive.

(4) What other aspects of our simulation do you think were not realistic? Brainstorm a list of possible inaccuracies or shortcomings of the model.

(5) Research facts that you need to know to address the shortcomings your group identified. In addition, determine a good estimate for the number of meadow voles a barn owl eats each month. Record what you find on a separate page. Remember to include the source of your information in case you need to find it again later.

(6) Based on the number of meadow voles a barn owl eats each month, what is the minimum number of meadow voles needed to sustain a family of four owls? (Use your chart on the previous page to estimate the value.)

Changing the Model

(7) Choose some aspect of the model that you would like to improve based on the facts you found earlier. Describe what you are trying to achieve with your new model.

(8) Figure out a way to implement your changes in the spreadsheet. Describe what you changed, and record the formulas you used. Choose several different values for the starting population of meadow voles, the number of owls, and other constants in your model. Try to choose one case where the owls eat all of the voles, one where the vole population stays approximately the same over time, and one where the vole population grows quickly. Graph each example. (9) For each of the three cases, state the values you used, summarize what happened, and include the corresponding graph. (10) Explain whether your model did what you hoped it would do. Do you think that this model is more realistic than our original model? Why or why not?

Instructor Notes

Owls and Voles Simulation

Have students wash their hands before beginning this activity if you are using food to represent the voles. Allow 15 to 30 minutes for running simulations. It will probably take students an additional 5 to 10 minutes to complete the table showing the minimum numbers of voles needed to sustain a family of owls.

Show the students a picture of a barn owl and explain that they will be playing the role of a family of barn owls. Show a picture of a meadow vole, the main food source for most barn owls. Explain that they will be working with a mathematical model of how owls and voles interact. The model is simplified from the way things work in real life, but has some realistic features.

Most groups end up eating too many voles the first time they try this game so that the family of owls goes extinct. This is ideal for the first round, and it is more likely to happen if you emphasize the importance of eating enough voles during the instructions.

If any families die out, ask them to explain what happened. Point out that they need a hunting strategy that lets them eat as much as possible over time, but that also leaves enough voles to reproduce and replace the ones they have eaten. Ask the students to discuss a strategy that will enable all of the owls to survive for 6 months. Once they have a strategy, have them begin a new simulation using the tally sheet in the next column.

Once the students have completed 2 to 4 simulations, they are ready to work on the table showing the minimum number of voles needed to feed a family of owls sustainably. You may choose to complete this table as a class. The answers are shown below.

Minimum Number of Voles Needed

Number of owls in the family	If each owl eats 1 vole	If each owl eats 2 voles	If each owl eats 3 voles	If each owl eats v voles
3	9	18	27	$9v$
4	12	24	36	$12v$
5	15	30	45	$15v$
n	$3n$	$6n$	$9n$	$3 \cdot n \cdot v$

Inaccuracies in the Model

Allow about 30 to 45 minutes for identifying inaccuracies, looking up facts on the internet, and coding up the original model. (The information gathering step could assigned as homework instead.)

Some inaccuracies in the model that students might notice include:

- Owls need to eat more than one vole each month. (They need about 6 each night.)
- Voles usually have more than one baby at a time. (Voles typically have 4-12 babies each month.)
- Voles born this month are not old enough to have babies the next month. (Voles take about 37 to 47 days to mature.)
- Owls do not confine their hunting to just one meadow. (The hunting range of a barn owl is about 1700 acres or 2.7 square miles.)

- Owls can eat other animals if the vole population gets low. (They can eat small lizards, birds, and other rodents.)
- Voles are eaten by other animals besides owls. (Foxes, hawks, crows, cats, and snakes are just some animals that eat voles.)
- Animals can also die of old age.
- A meadow can only support so many voles before they eat up all the grass, seeds, and other plants. The level of the vole population that a meadow can support is called its *carrying capacity*. (There can be anywhere from 15 to 500 meadow voles per acre, with 50 being a typical carrying capacity).
- Our model does not account for the fact that owls must hunt for the voles.
- Our model does not account for the fact that it is more difficult for owls to find voles during the winter.

Note: Barn owls do reproduce once every six months if conditions are favorable, so that aspect of the model is somewhat realistic. Owls typically lay 6 to 12 eggs.

Owls must eat about 6 voles each night in order to survive. This means that each owl needs to eat about 180 voles per month. Using the formula we found earlier, we see that a family of 4 barn owls would need at least 2,160 voles in the hunting territory in order for their food supply to be replenished.

This rough estimate shows that it takes a LARGE population of voles to support a small number of owls. This illustrates why predators are almost always much less numerous than the animals they eat.

To implement the model in the spreadsheet program, students could complete the following steps. It is possible to show fewer columns than I list here by combining them. I like using separate columns because it keeps the various influences separate and it is easier to see what is happening.

- Make column headings for month, vole population, voles eaten, and voles born.
- Fill in month numbers from 0 up to at least 24 months.
- Start the vole population in cell B2 at 2,160.
- The number of voles eaten each month by four owls is $180 \cdot 4 = 720$. You can just copy this value down for the whole column.
- To find the number of voles born, we need to find half of the voles that were not eaten. We need to use the floor function to make sure that we get a whole number. To find this value, type "=FLOOR(1/2*(B2-C2),1)" for the first row.
- To find the vole population for the next row, use the formula "=B2-C2+D2".
- Copy the lowest cell in each column into the lower cells.

Changing the Model

Allow 30 to 60 minutes for students to consider how they would like to change the model and to implement their changes.

Here are some ways that students might modify the simple model to make it more realistic. Some of these modifications are quite simple, while others can get extremely complicated.

- We could estimate that each meadow vole female produces about 6 baby voles each month. This would change the formula in the column for voles born to "=FLOOR(6 * 1/2 *(B2-C2),1)". The 1/2 in the original formula is a way of estimating that about half of the voles are female. We multiply this number of females by 6.
- Suppose we want to account for the fact that voles are not old enough to have babies the next month, but we want to allow them to have babies once they are two months old. This means that we should not add the baby voles into the population until two rows after the row when they are born. To implement this, fill in cells B2, C2, and D2 as described before. (You will need to increase the starting value of B2, eventually). Next, in cell B3, type the formula "=B2-C2". Column C will be filled in with the number of voles eaten each month just as before. Column D can be copied from cell D2 as before. Cell B4 will use the formula "=B3-C3+D2". Copy this formula into all of the other cells in column B.
- Suppose we want to let some of the voles die of old age or other causes. Voles typically live for about 16 months in the wild, so we could assume that about 1/16 of the voles die every month of old age. We could add another column to our spreadsheet that computes 1/16 of the vole population. We could subtract this from the total population either before or after computing the number of voles born.
- Accounting for the carrying capacity makes things a bit more complicated. Suppose that the carrying capacity of the fields in the owl family's hunting range is about $1700 \cdot 50 = 85,000$ meadow voles. We might suppose that when the vole population is much lower than this value that the owls have trouble finding voles to eat and eat other things instead. When the vole population gets closer to this value or goes above this value, the owls should have a much easier time finding voles to eat. There are many ways to put this idea into a model. One way is to double the baseline number of voles eaten, multiply by the current vole population, V_{new}, and then divide by 85,000. The fraction $V_{\text{new}}/85,000$ will get larger as the population gets close to the carrying capacity. The doubling represents the assumption that an owl could eat at most 12 voles (instead of the usual 6 voles) per night.
- Suppose that we would like to include the fact that voles will die off if there are too many voles for the fields to support. One way to do this is to modify the death term of 1/16 that we included above. We could add, for example, 1/4 of $V_{\text{new}}/85,000$ to the 1/16 before multiplying by the population V_{new}. In this way, when the vole population is close to 85,000 a larger fraction of voles will die. When the vole population is far away from 85,000 a smaller fraction of voles will die. I chose the 1/4 as a rough guess that about 1/4 of the population might die when the population reaches the carrying capacity. A mathematical biologist might try to refine this parameter by comparing the model with population data.
- It is possible to include columns for the owl population as well. These columns can account for the birth of baby owls and the death of owls due to a variety of causes. Owls are more likely to die when it is hard to find voles to eat.

Conclusion

Allow 15 to 45 minutes for the conclusion.

Ask the students to share their results.

Discuss the role that models play in science. Ask students why a scientist might use a mathematical model rather than simply gathering data and seeing what happens. A few reasons that scientists use mathematical models are to predict what will happen if current conditions change, or to decide whether a few simple factors are enough to explain what they observe in the real world.

Ask students why they think that scientists might sometimes prefer a more complicated model rather than a simple model. (A complicated model can include more details about the situation.) Why do they think that scientists sometimes prefer to use the simpler model rather than the more complicated one? (Sometimes the details are not important for understanding the question. Sometimes it is too difficult to figure out what is happening with the more complicated model and it can be helpful to tackle the simple model first.)

At a conference at the New Jersey Institute of Technology in May 2007, mathematical biologists presented many mathematical models similar to the one in this activity. A few of the topics were:

- The effects of sunlight on two competing species of Daphnia (water fleas).
- A three-stage life cycle model for the green treefrog.
- Understanding how wading birds in the New York Harbor choose islands for nesting each year.
- The ecological impact of edge effects resulting from clear-cutting in tropical forests.
- Evaluating methods for preventing the spread of a tree disease epidemic.
- The spread of cancer cells and tumor growth.

Students who enjoyed this activity might like to try an independent research project on a similar topic.

Taking it Further

There are several ways to build this activity into a more extensive unit.

You may wish to have students dissect owl pellets before or after this activity. Owl pellets are available from science stores online and typically cost about $2 or $3 apiece. Most kits include bone charts to help students identify the animals in the pellets.

This activity could easily be extended into a population modeling unit that investigates the behavior of various linear equations and other functions. The graphical technique called "cobwebbing" brings in graphing concepts and is a nice visual way to show what happens to a population for different starting conditions.