A note about music ...

The following is the text of a MATLAB program that produces a sound and a graph of the sound wave. In this case it is a Pythagorean third:

third.m

```
function [t,y]=third
t=0:1/8192:8;
y = 1/2*(sin(2*pi*440*t)+sin(2*pi*5*110*t));
soundsc(y)
figure('color','white')
plot(t(1:128),y(1:128))
titlestr = 'third';
title(titlestr,'FontSize',14)
wavwrite(y,'third')
```

The following graphs display the 440 pitch and the 440 pitch with a harmonic third overtone:

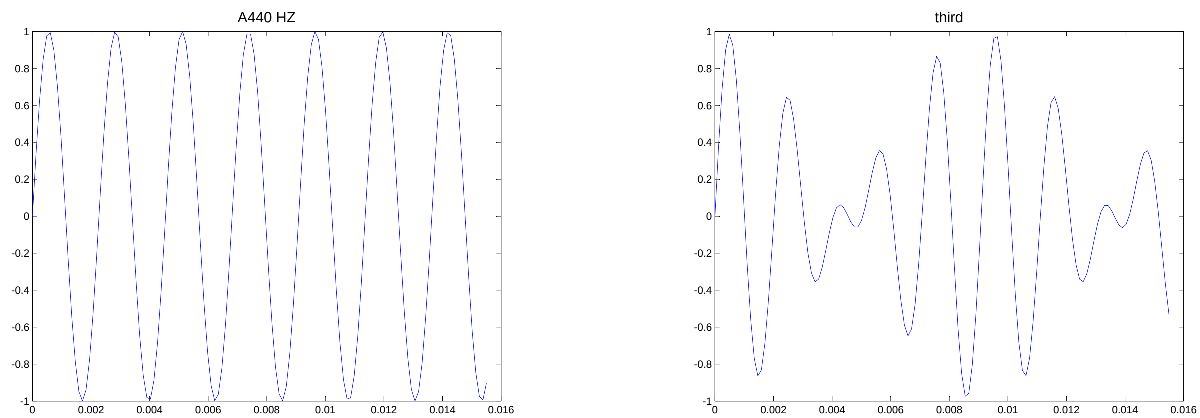

Figure 1: Waveforms for 440 and the A-major third.

1. In Pythagorean tuning all frequencies used are multiples of $2^n 3^m$. One can start with the note of F and then increase by musical fifths so $3/2$ to get other notes. The next note will be a C. By moving notes up and down octaves one may be sure that all of the notes will fit between this C and the next C. The result is the C-major scale:

note	C	D	E	F	G	A	B	C
frequency ratio	1	$\frac{9}{8}$	$\frac{81}{64}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{27}{16}$	$\frac{243}{128}$	2

   In five-limit just intonation all frequencies used are multiples of $2^n 3^m 5^p$. Find the five-limit approximations to the Pythagorean frequency ratios that have the smallest denominators.

2. We saw that five-limit tuning led to the syntonic comma $81/80$. Pythagorean tuning leads to the Pythagorean comma. Take an educated guess of the value of this comma by comparing the powers of 2 to the powers of 3.

3. Given that the international standard for A4 is 440, and that A4 is the A between C4 and C5, compute the frequencies of each note in the C major scale using 12 tone equal temperament tuning.

4. We arrived at the 12TET semitone ratio via a rational approximation to $\log_2(3)$. Use the same procedure to derive the first four rational approximations to π, and the first four rational approximations to $\sqrt{3}$.