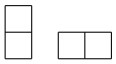
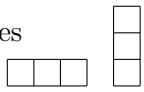
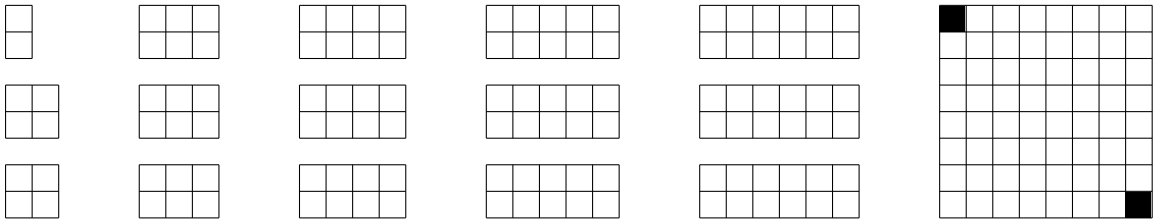
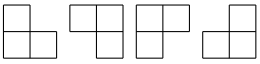
ARIZONA STATE UNIVERSITY

Julia Robinson Mathematics Festival

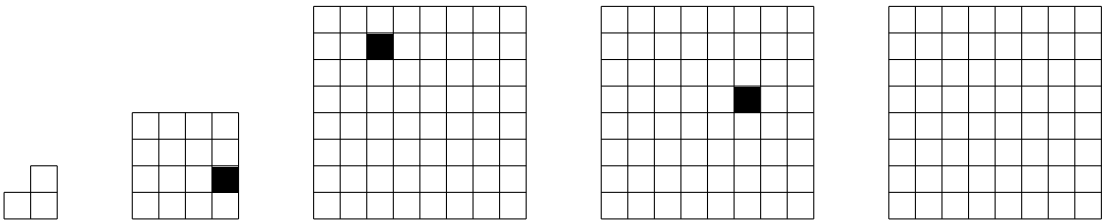
Tilings with dominos, straight, bent and triangular triominos

1. In how many different ways can a rectangular $2 \times n$ board be tiled with dominos? Start with small numbers n, then generalize.
2. Is it possible to tile a 5×5 square board with dominos? 
3. Is it possible to tile with dominos a 5×5 square board from which one square has been removed? Does it matter which one has been removed?
4. Is it possible to tile with dominos an 8×8 rectangular board from which two opposite corner have been removed?
5. Find all squares on an 8×8 rectangular board such that if one of these squares is removed, then the remaining part can be tiled with (straight) triominos. 

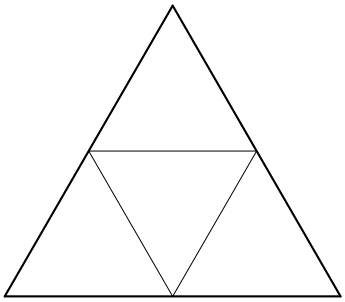
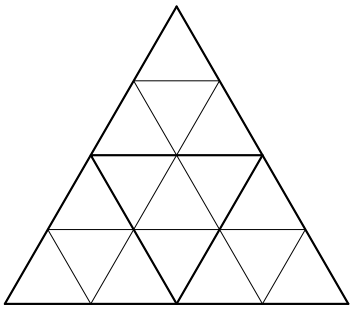
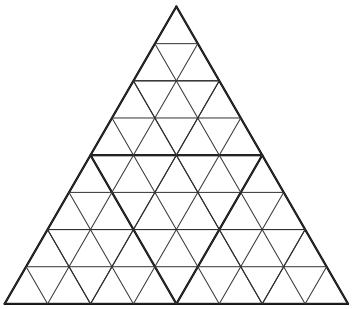


A bent triomino is of a 2×2 board from which one corner has been removed. 

6. Is it possible to tile an $2^n \times 2^n$ board from which one square has been removed with bent triominos? Does it matter which square has been removed?
7. Which square boards of size $n \times n$ from which one square has been removed can be tiled with bent triominos? (Hint: The 5×5 board is special.)
8. Which $n \times m$ rectangular boards can be tiled with bent triominos?



Write $T(n)$ for a triangular board of side-length 2^n which is subdivided into equilateral triangles each of side length 1. If a triangle shares one (or two) of its sides with the large triangle, then it is called an *edge triangle*. If it shares two of its sides with the large triangle, then it is called a corner triangle. A (triangular) triomino is a *tile* consisting of three adjacent triangles. 

9. For which n is it possible to tile the remaining board with triangular triominos after any (one) corner triangle is removed from $T(n)$?
10. For which n is it possible to tile the remaining board with triangular triominos after any (one) edge triangle is removed from $T(n)$?
11. For which n is it possible to tile the remaining board with triangular triominos after all the corner triangles and any other (one more) triangle are removed from $T(n)$?
12. For which n is it possible to tile the remaining board with triangular triominos after if any (one) triangle not adjacent to a corner triangle is removed from $T(n)$?